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INVESTIGATION INTO THE REINFORCEMENT PRINCIPLES FOR SKEWED BRIDGE REINFORCED CONCRETE SLAB ELEMENTS

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Abstract. *The paper addresses an urgent issue arising in the design of reinforced concrete slabs for skew bridges and overpasses with a parallelogram shape in plan. In this paper, the reinforcement principles for this type of slabs are investigated. It is noted that relevant guidelines on this matter are practically absent in the current regulatory documents; therefore, the aim of this study is to develop the missing design recommendations. The research was carried out using numerical modeling based on the Finite Element Method (FEM) in the "LIRA-SAPR" software package, as well as using the Wood-Armer non-orthogonal analytical method. A comparative analysis of the analytical calculation results and the reinforcement fields (mosaics) obtained in "LIRA-SAPR" shows that at skew angles up to α=15°, both traditional orthogonal and fan-shaped reinforcement methods are equally expedient. However, within the skew angle range of 15° < α < 60°, the only effective engineering solution is the exclusive use of the fan-shaped reinforcement layout. Fan-shaped reinforcement represents an approach that not only corresponds to the actual stress-strain state of the structure but is also economically efficient.*

Keywords: *skew bridges, non-orthogonal reinforcement, Wood-Armer method, LIRA SAPR, torsional moment, reinforced concrete, finite element method.*

1. INTRODUCTION

In modern urban planning and transport infrastructure construction, there is a growing demand for the design of bridges and overpasses that cross obstacles at non-right angles. In such cases, the slab elements of the structure take on a parallelogram shape in plan. Typically, when reinforcing these types of slabs, the rebar layout is arranged parallel to the sides of the slab (Fig. 1). As a result, a non-orthogonal angle is formed between the axes of the reinforcement mesh. Most modern structural analysis software packages based on the Finite Element Method (FEM) automatically perform reinforcement selection strictly within an orthogonal coordinate system. Thus, a dangerous scientific and practical deadlock occurs: the digital model is "unaware" that the mesh is non-orthogonal, while the actual rebar layout laid out at an angle loses its design efficiency, putting the structural safety of the facility at risk.

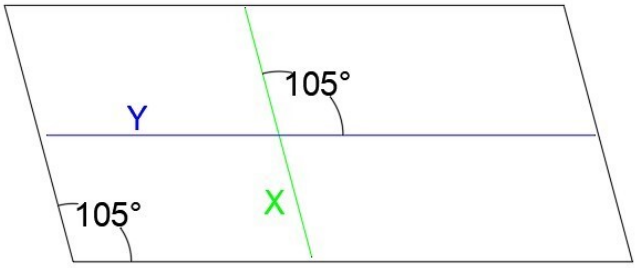


Figure 1. Non-orthogonal reinforcement of a slab with a 15° skew

2. METHODOLOGY AND MATHEMATICAL APPARATUS

When designing skewed reinforced concrete bridge slab elements, the axes of the working reinforcement mesh often do not coincide with the direction of principal stresses. In the case of non-orthogonal reinforcement, where the X and Y axes intersect at an angle α (Fig. 1), the internal bending and twisting forces (M_x, M_y, and M_{xy}) obtained from standard linear-elastic Finite Element Method (FEM) calculations must be transformed into equivalent ultimate resistance moments (M^*_x, M^*_y). This transformation process is based on the Wood-Armer analytical model. The fundamental objective of the method is to ensure load-bearing capacity at every point of the structure within the principles of limit equilibrium. [2]

$$M^*_y = M_y + 2M_{xy}cot\alpha + M_x cot^2\alpha + \left|\frac{M_{xy} + M_x cot\alpha}{sin\alpha}\right| \quad (1)$$

$$M^*_x = \frac{M_x}{sin^2\alpha} + \left|\frac{M_{xy} + M_x cot\alpha}{sin\alpha}\right| \quad (2)$$

The internal ultimate resistance moment of the non-orthogonal reinforced concrete mesh must exceed the external moment generated by actions acting in the plane of any potential inclined crack that may form in the concrete.

The percentage difference in design moments arising between the standard orthogonal calculation method and the proposed Wood-Armer non-orthogonal analytical model for different skew angles α is presented in Table 1. The results in Table 1 clearly demonstrate that as the angle between the reinforcement axes increases, the percentage increase in the required reinforcement area exhibits a non-linear behavior. Even at minor skews where α=105°, 36.1% more working reinforcement is required compared to the standard orthogonal calculation method. When the skew angle α exceeds 110°, this discrepancy rises to +53.8% along the X direction. At an angle of α=120°, this error reaches a critical threshold, significantly increasing to 111.3%.

Table 1

Angle between X and Y reinforcement (α)	Along X-direction	Along Y-direction
α= 105°	+36.1%	+15.0%
α= 110°	+53.8%	+22.4%
α= 120°	+111.3%	+46.4%

3. NUMERICAL EXPERIMENT AND RESULTS

As part of the numerical experiment, calculations were carried out using the “LIRA-SAPR” software. The obtained results are displayed below in the form of reinforcement layout fields (mosaics) (Fig. 2).

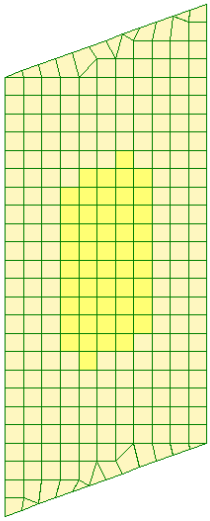


Figure 2a. At skew angle $\alpha < 15°$

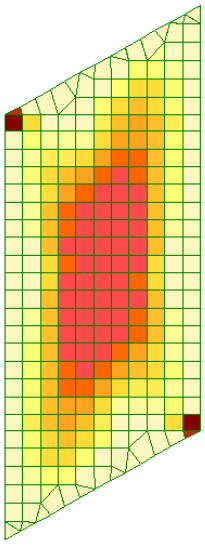


Figure 2b. At skew angle $15° < \alpha < 60°$

To determine internal forces in skew slabs, the classical Wood-Armer method adds the absolute value of the twisting moment to the bending moments in both directions. Although this approach creates a reliable safety margin to prevent cross-sectional failure, it leads to severe economic and structural disadvantages in the design of skew slabs. As the skew angle increases, intense torsional effects occur in the slab. Because the Wood-Armer equations attempt to distribute these stresses strictly along the predefined rectilinear axes, the reinforcement area required by the calculation is artificially inflated. The wide red area visible in Figure 2b is direct evidence of the non-optimal material consumption caused by the Wood-Armer method. A sharp increase in reinforcement not only raises the estimated construction cost but also leads to over-reinforcement of the section, thereby inducing a risk of brittle concrete failure [1,2].

The fan-shaped reinforcement method proposed in this study solves this problem fundamentally. Since the rebars are placed along the actual trajectories of tensile stresses, the twisting moment in the coordinate system drops to a minimum. Consequently, the artificial reinforcement increase brought by the Wood-Armer method is eliminated, and material consumption is optimized.

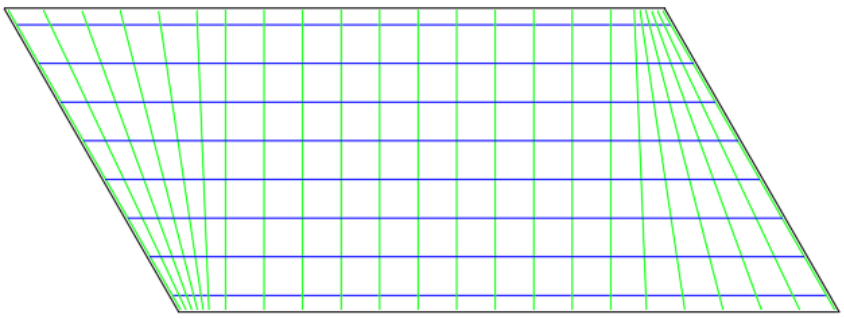


Figure 3. Fan-shaped reinforcement along stress trajectories

An analysis of the structural layout presented in Figure 3 shows that aligning the reinforcement bars with the trajectories of the developing stresses is the most effective engineering solution for optimizing material consumption. Unlike non-orthogonal (skewed) meshes, the placement of the working reinforcement in this model coincides with the axes of the principal bending moments. Thanks to this geometric alignment, there is no need to artificially increase the required reinforcement area. Since the rebar layout is not designed within a non-orthogonal coordinate baseline, the mathematical equivalents of moment amplification arising in the Wood-Armer equations become zero. From a mechanical standpoint, the reinforcement cage operates with the highest efficiency factor against internal forces, which prevents the necessity of placing extra rebars in the corners, reduces material consumption, and ensures structural reliability [1,2].

CONCLUSIONS AND RECOMMENDATIONS

1) When the skew angle is up to $\alpha=15°$, the stress-strain state of the slab does not differ significantly from a conventional rectangular slab. Within this range, the application of both the traditional orthogonal and the proposed fan-shaped reinforcement layouts is equally effective and yields similar results;

2) When the skew angle is greater than $\alpha=15°$ and particularly up to $\alpha=60°$, the implementation of the fan-shaped reinforcement method becomes essential;

3) Alignment with actual stresses: Reinforcement bars are distributed along the real trajectories of tensile forces developing within the slab;

4) Mitigation of torsional effects: Due to the dense arrangement of bars at the acute corners, intense twisting moments are minimized;

5) High economic efficiency: Significant savings in structural steel reinforcement are achieved without reducing the safety and load-bearing capacity of the structure.

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