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NUMERICAL ANALYSIS OF UNSTEADY LIQUID-PHASE FLOW IN A PIPELINE

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Abstract. A numerical analysis of unsteady liquid-phase flow in a pipeline is presented using a modal representation of the solution. The temporal coefficients of the Fourier series are governed by a system of ordinary differential equations obtained from the original mathematical model after linearization and application of the Laplace transform. In the numerical example, the characteristic time of the liquid phase is defined independently as $\tau_2=l/c_2$ and, for $l=1000$ m and $c_2=1000$ m/s, equals 1 s. The first modal pressure component, the spatial pressure distribution, the mass-flow-rate change, and the convergence with respect to the number of harmonics are investigated. Calculations for N=1, 3, 5, 10, 100, and 200 show that the solutions for N=100 and N=200 are practically identical. The maximum amplitude of the first pressure harmonic is approximately 2.62 MPa. The results demonstrate that accounting for the liquid phase's own time scale has a substantial effect on the transient dynamics.

Keywords: *liquid phase, pipeline, transient process, Laplace transform, Fourier series, mass flow rate, convergence*

Introduction. Transient processes in pipeline systems are accompanied by changes in pressure, velocity, and mass flow rate of the transported medium. In multiphase systems, an additional difficulty arises from differences in the physical properties of the phases and, consequently, from different disturbance-propagation velocities. Classical one-dimensional and two-fluid models are widely used to analyze such processes [1–4]. In the study of transient regimes, it is important not only to obtain an analytical form of the solution but also to assess its numerical stability and convergence when the modal series is truncated [4–6].

In the present work, the liquid phase is extracted from the general gas-liquid flow model and analyzed numerically as an independent subsystem. Particular attention is paid to the correct choice of the characteristic time $\tau_2=l/c_2$, calculation of the temporal coefficients $\psi_{2i}(t)$, reconstruction of the pressure field $P_2(x,t)$, evaluation of the mass-flow-rate change, and verification of solution convergence as the number of harmonics increases.

1. Problem Formulation and Computational Model. The liquid-phase pressure distribution is represented as the sum of a linear component satisfying the boundary conditions and a Fourier series:

$$P_2(x,t) = P_1(t) - \left[\frac{P_1(t) - P_2(t)}{l}\right] \cdot x + \sum \psi_{2i}(t) \cdot \sin\left(\frac{i\pi x}{l}\right)$$

For each harmonic, the temporal coefficient satisfies a second-order differential equation

$$\ddot{\psi}_{2i} + 2h_{2i}\,\dot{\psi}_{2i} + n_{2i}^2\,\psi_{2i} = F_{2i}(t)$$

where h_{2i} is the damping coefficient, n_{2i} is the natural frequency, and $F_{2i}(t)$ is the external forcing function determined by changes in the boundary pressures. To make the notation consistent with the inverse Laplace-transform form, the damped frequency is introduced as

$$\omega_{2i}^2 = n_{2i}^2 - h_{2i}^2$$

In the numerical calculation, the second boundary pressure is assumed constant, while the first is prescribed by a smooth cosine function that ensures zero derivatives at the ends of the transient interval:

$$P_1(t) = P_{20} + \frac{P_{10} - P_{20}}{2} \cdot [1 + cos(\frac{\pi t}{\tau_2})], \quad 0 \le t \le \tau_2$$

$$P_2(t) = P_{20} = const, \qquad \tau_2 = \frac{l}{c_2}$$

With this specification, the conditions $P_1(0)=P_{10}$, $P_1(\tau_2)=P_{20}$, and $\dot{P}_1(0)=\dot{P}_1(\tau_2)=0$ are satisfied. The change in total mass flow rate is obtained using the pipe cross-sectional area $S=\pi r^2$ and the relation $\Delta\dot{m}_2=\Delta Q_2 S$.

2. Input Data and Numerical Procedure

Parameter	Value	Unit
l	1000	m
c_2	1000	m/s
ρ_2	800	kg/m³
a_2	0.1	s^{-1}
V_{20}	0.35	m/s
P_{10}	10	MPa
P_{20}	5	MPa
K	$1 \cdot 10^{-9}$	m²/N
D	0.3	m²/s
r	0.15	m

Table 1. Computational parameters of the liquid phase

For the adopted parameter set, $\tau_2=l/c_2=1$ s. The value $V_{20}=0.35$ m/s is taken as a working assumption $V_{20}=V_{10}$ for the preliminary numerical analysis and should be refined when technological or experimental data are available. The solution was calculated for N=1, 3, 5, 10, 100, and 200 harmonics.

3. Results and Discussion.The first harmonic $\psi_{21}(t)$ starts from zero with a zero initial derivative, in agreement with the adopted initial conditions. For $\tau_2=1$ s, the maximum absolute amplitude of the first harmonic is approximately 2.62 MPa. A change in the sign of $\psi_{21}(t)$ does not imply negative absolute pressure, because this function is a modal correction to the baseline pressure distribution.

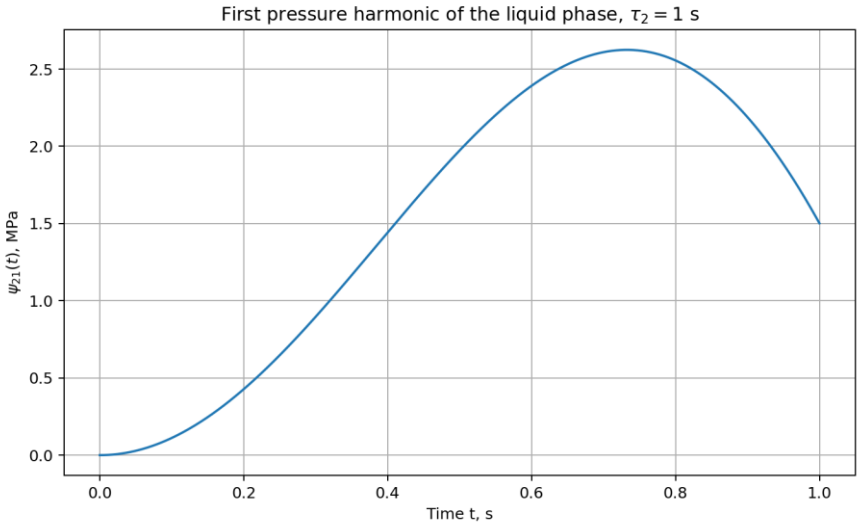


Figure 1. First liquid-phase pressure harmonic at τ_2=1 s

The spatial pressure distribution at several time instants is shown in Fig. 2. Throughout the considered interval, the absolute pressure remains positive and the boundary values satisfy the prescribed conditions. Local extrema inside the pipeline arise from the contribution of transient harmonics and characterize the transient nature of the process.

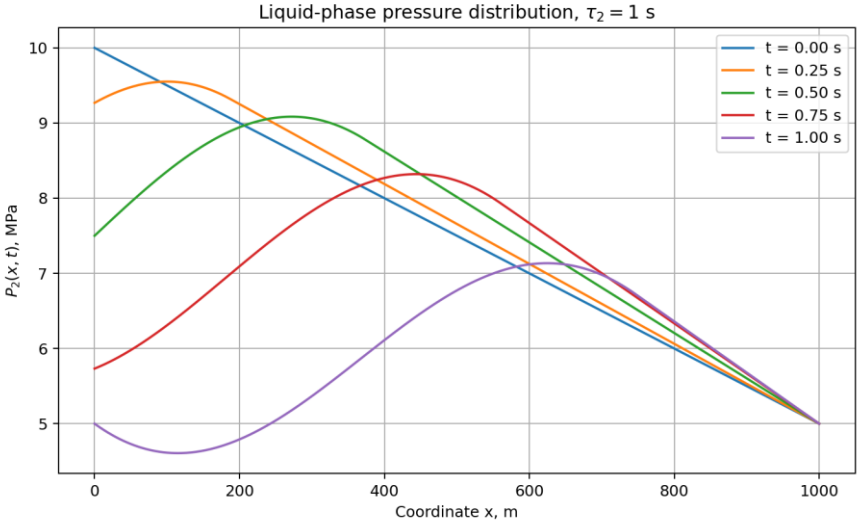


Figure 2. Liquid-phase pressure distribution along the pipeline

To assess accuracy, a convergence study was carried out for N=1, 3, 5, 10, 100, and 200. Increasing the number of harmonics stabilizes the spatial pressure profile. The maximum deviation of the N=100 solution from the N=200 result over the entire computational domain is approximately 0.265 kPa, while at $t=\tau_2/2$ it is about 0.069 kPa. Thus, for the considered parameter set, N=100 provides practically the same result as N=200.

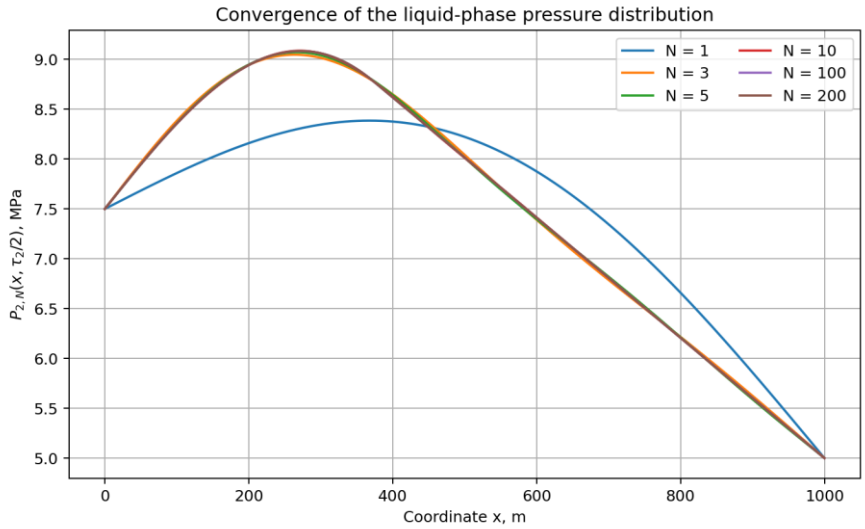


Figure 3. Convergence of the liquid-phase pressure distribution

The mass-flow-rate change at the pipeline outlet exhibits a pronounced transient behavior. For N=200, the calculated values are $\Delta\dot{m}_2 \approx -5.60$ kg/s at t=0.25 s, 14.92 kg/s at t=0.50 s, 48.32 kg/s at t=0.75 s, and 73.87 kg/s at t=1.00 s. The negative value at the initial stage does not represent a negative absolute flow rate: $\Delta\dot{m}_2$ is the change relative to the inlet value.

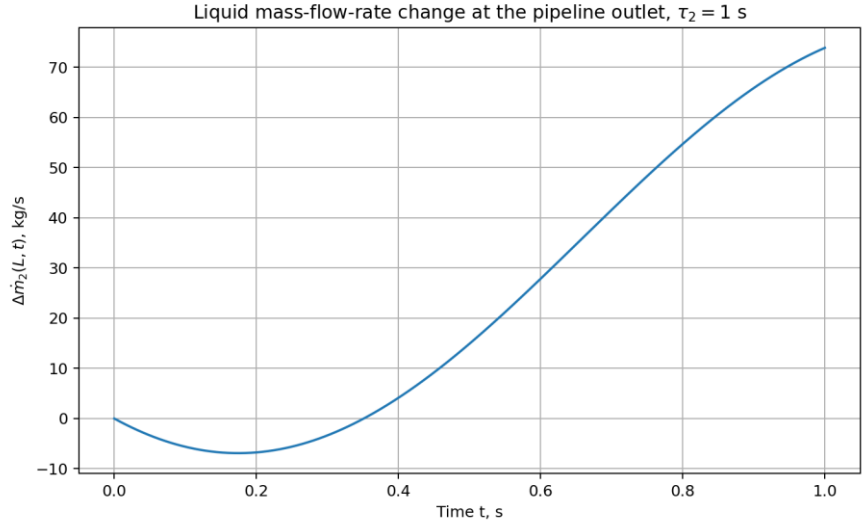


Figure 4. Liquid mass-flow-rate change at the pipeline outlet

The results show that using the liquid phase's own characteristic time substantially changes the scale of the transient process. With τ_2=1 s, the pressure and flow-rate dynamics develop faster than when the gas-phase time scale is used. At the same time, the convergence check up to N=200 confirms the stability of the modal representation and supports the use of approximately N=100 in subsequent parametric studies without a noticeable loss of accuracy.

Conclusion. An independent numerical analysis of the liquid phase in a transient pipeline process has been performed. The characteristic time is defined as $\tau_2 = l/c_2$ and equals 1 s for the selected parameters. Based on the modal solution, the time dependence of the first pressure harmonic, the spatial pressure distribution, and the outlet mass-flow-rate change were calculated.

The maximum amplitude of the first modal pressure component is about 2.62 MPa. The solution maintains positive absolute pressure over the entire considered range. Convergence of the series was confirmed by calculations up to N=200; the difference between N=100 and N=200 is small. The obtained dependencies can be used for further sensitivity analysis of the model with

respect to K, D, V_{20}, c_2, and the boundary-pressure difference. For final engineering interpretation, the initial liquid-phase velocity V_{20} should be refined using experimental or technological data.

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