



STABILITY OF CYLINDRICAL COATING MADE OF POROUS MATERIAL

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Abstract: The presented article considers the issue of loss of strength of a thin-walled cylindrical coating made of porous material rigidly bonded at the ends because of thermal expansion. It is necessary to find the critical temperature value necessary for loss of stability. The variational principle of analytical mechanics was used to solve the problem. From the condition of stationarity of the variational principle, a system of differential equations and initial conditions are obtained. The differential equations of motion within the found initial conditions were performed numerically. By the nature of the equation, the fourth-order Runge-Kutta method was chosen. For Astm 316 and SW 316 steels used in nuclear reactors, they were solved numerically on a computer. The results are displayed as graphs and have been analyzed.

Keywords: *Porous material, cylindrical coating, loss of stability, variational principle, Runge-Kutta method.*

1. INTRODUCTION

It is known that in nuclear power plants, the main element is fuel cells (HNE). The elements that generate heat consist of a core made of the element uranium, a steel cylinder holding it inside, and liquid metal flowing through their intermediate space. Due to the fast neutrons emitted by the uranium core, the cylindrical element receives a large dose of radiation, and due to this, a very large amount of heat is released. Since the heat capacity of the water vapor is not sufficient to conduct such a large amount of heat, the liquid metal is discharged from the intermediate gap. Despite the fact that the cylindrical shell is made of steel material, on the one hand, it expands due to the heat released, and on the other hand, due to the collision of fast neutrons from the uranium core with the atoms of the cylinder, the atoms leave their places. The atoms that have left their place shoot out and displace other atoms from their places. After several collisions, the energy of the atom decreases, and as a result they get stuck between other atoms. Leaving the atoms in their

place leads to rarefaction in this place, and riveting leads to condensation. Thus, structural changes occur in the cylinder material, and the material behaves like a porous material. In space rockets and jet planes, the nozzles have a round cylindrical shape. It gets very hot when ignited fuel comes out of the nozzle. They make nozzles consisting of several layers so that the heat removed is not transferred to the rocket or aircraft. Some of these layers are made of porous material. Let us choose a coordinate system so that the origin is located above the middle surface of the cylinder at the same distance from both ends. Let us assume that the X-axis is directed along the cylinder axis, the Y-axis is perpendicular to the X-axis in the direction touching the middle surface of the cylinder, and the Z-axis is directed in the normal direction of the surface.

Bundan başqa aşağıdakı fərziyyələri qəbul edək.

- 1) Kirxof-Ləv hipotezi ödənilir, yəni deformasiya olunana qədər silindrin doğrurana perpendikulyar müstəvi kəsiklər deformasiyadan sonra da doğrurana perpendikulyar olub, müstəvi qalır.
- 2) Bütün nöqtələr ancaq z oxu istiqamətində yerdəyişmə alır, bu yerdəyişmələr ancaq x dəyişənindən aslıdır (Məsələ oxa simmetrik olduğundan y oxundan asılılıq yoxdur. Qalınlıq kiçik olduğundan yerdəyişmənin z dəyişənindən asılılığı da nəzərə alınmır).
- 3) Bütün deformasiya və gərginliklər z dəyişənindən xətti asılıdırlar. Silindrin qalınlığını $2h$, uzunluğunu $2l$ ilə işarə etsək, $-h \leq z \leq h$; $-l \leq x \leq l$.

If we denote the offset in the direction of the X-axis with the letter u , the offset in the direction of the Y-axis with the letter v , and the offset in the direction of the Z-axis with the letter w , we get $u = 0; v = 0$.

It is known that the coefficients of the second quadratic form for a round cylindrical surface are as follows:

$$b_{11} = b_{12} = b_{21} = 0; b_{22} = \frac{1}{R}. \quad (1)$$

Where R - is the radius of the average surface of the cylinder.

According to the boundary conditions

$$w(x, T)|_{x=-l} = 0, \quad \frac{\partial w(x, T)}{\partial x} \Big|_{x=l} = 0 \quad (2)$$

Where T is the temperature.

(2) Let's choose the following approximation for $w(x, T)$ according to the boundary conditions and the physical representation:

$$w(x, T) = f(T) \cdot \left(\cos \frac{\pi x}{\ell} + 1 \right). \quad (3)$$

Where $f(T)$ is an unknown function of temperature.

If we consider the geometric nonlinearity only in the w direction, then the dependence of the physical components of the strain tensor on the components of the displacement vector will look like this:

$$\begin{cases} \varepsilon_{11} = \varepsilon_{xx} = \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2; \dot{\varepsilon}_{11} = \dot{\varepsilon}_{xx} - \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_{22} = \varepsilon_{\varphi\varphi} = \frac{w}{R}; \dot{\varepsilon}_{22} = \dot{\varepsilon}_{\varphi\varphi} = 0 \end{cases} \quad (4)$$

The physical relations defined by the relations (3) have the following form

$$\begin{aligned} \dot{\varepsilon}_{xx}^e &= \frac{1}{E} \left[\dot{\varepsilon}_{xx}^v + \nu \dot{\varepsilon}_{\varphi\varphi}^v \right] \\ \dot{\varepsilon}_{\varphi\varphi}^e &= \frac{1}{E} \left[\dot{\varepsilon}_{\varphi\varphi}^v + \nu \dot{\varepsilon}_{xx}^v \right] \\ \dot{\varepsilon}_{xx}^v &= \frac{1}{3} \left(\dot{\varepsilon}_{xx}^e + \dot{\varepsilon}_{\varphi\varphi}^e + \dot{\varepsilon}_{\varphi\varphi}^v \right) \\ \dot{\varepsilon}_{\varphi\varphi}^v &= \frac{1}{3} \left(\dot{\varepsilon}_{xx}^e + \dot{\varepsilon}_{\varphi\varphi}^e + \dot{\varepsilon}_{xx}^v \right) \end{aligned}$$

Where S is the change in relative volume due to temperature alone.

Taking into account the last relations (2), the equations of the velocity case are reduced to the following form

$$\begin{cases} \dot{\varepsilon}_{xx}^e = \frac{1}{E} \left[\dot{\varepsilon}_{xx}^v + \nu \dot{\varepsilon}_{\varphi\varphi}^v \right] + \frac{1}{3} \dot{\varepsilon}_{xx}^v \\ \dot{\varepsilon}_{\varphi\varphi}^e = \frac{1}{E} \left[\dot{\varepsilon}_{\varphi\varphi}^v + \nu \dot{\varepsilon}_{xx}^v \right] + \frac{1}{3} \dot{\varepsilon}_{\varphi\varphi}^v \end{cases} \quad (5)$$

We solve the system (5) with respect to voltage. If we multiply each, part of the first equation by E and the second by νE and add it from side to side:

$$\dot{\varepsilon}_{xx}^e = \frac{E}{1-\nu^2} \left(\dot{\varepsilon}_{xx}^v + \nu \dot{\varepsilon}_{\varphi\varphi}^v \right) + \frac{ES}{3(1-\nu)}. \quad (6 a)$$

If we multiply each, part of the first equation by νE and the second by E and add it from side to side:

$$\dot{\varepsilon}_{\varphi\varphi}^e = \frac{E}{1-\nu^2} \left(\dot{\varepsilon}_{\varphi\varphi}^v + \nu \dot{\varepsilon}_{xx}^v \right) + \frac{ES}{3(1-\nu)}. \quad (6 b)$$

If we decompose the stresses in the order of z , being satisfied with linear limits:

$$\left\{ \begin{array}{l} \tilde{\epsilon}_{xx} = \frac{1}{2h} r_{xx} + \frac{3z}{2h^3} M_{xx} \\ \tilde{\epsilon}_{\varphi\varphi} = \frac{1}{2h} J_{\varphi\varphi} + \frac{3z}{2h^3} M_{\varphi\varphi} \end{array} \right. \quad (7)$$

Burada,

$$N_{xx} = \int_{-h}^h \tilde{\epsilon}_{xx} \quad N_{\varphi\varphi} = \int_{-h}^h \tilde{\epsilon}_{\varphi\varphi} \quad M_{xx} = \int_{-h}^h z \tilde{\epsilon}_{xx} \quad M_{\varphi\varphi} = \int_{-h}^h z \tilde{\epsilon}_{\varphi\varphi}$$

If we take into account the equalities (7) in the equalities (6), then the free limits and the coefficients of z are equal to each other:

$$\left\{ \begin{array}{l} N_{xx} = \frac{2hE}{1-\nu^2} (\epsilon_{xx} + \nu \epsilon_{\varphi\varphi}) - \frac{2hES}{3(1-\nu)} \\ N_{\varphi\varphi} = \frac{2hE}{1-\nu^2} (\epsilon_{\varphi\varphi} + \nu \epsilon_{xx}) - \frac{2hES}{3(1-\nu)} \\ M_{xx} = \frac{2h^3E}{3(1-\nu^2)} \kappa_{xx}; \quad M_{\varphi\varphi} = \frac{2h^3\nu E}{3(1-\nu^2)} \kappa_{xx} \end{array} \right. \quad (8)$$

From (3)

$$\frac{\partial w}{\partial x} = -\frac{\pi f}{l} \sin \frac{\pi x}{l}; \quad \frac{\partial^2 w}{\partial x^2} = -\left(\frac{\pi}{l}\right)^2 f \cos \frac{\pi x}{l}$$

Considering these equalities in (4):

$$\left\{ \begin{array}{l} \epsilon_{xx} = \frac{1}{2} \left(\frac{\pi}{l}\right)^2 f^2 \sin^2 \frac{\pi x}{l}; \quad \kappa_{xx} = \left(\frac{\pi}{l}\right)^2 f \cos \frac{\pi x}{l} \\ \epsilon_{\varphi\varphi} = \frac{f}{R} \left(\cos \frac{\pi x}{l} + 1 \right); \quad \kappa_{\varphi\varphi} = 0 \end{array} \right. \quad (9)$$

Considering (9) - u in (8):

$$\begin{aligned} N_{xx} &= \frac{2hE}{1-\nu^2} \left[\frac{1}{2} \left(\frac{\pi}{l}\right)^2 f^2 \sin^2 \frac{\pi x}{l} + \frac{\nu f}{R} \left(\cos \frac{\pi x}{l} + 1 \right) \right] - \frac{2hES}{3(1-\nu)} \\ N_{\varphi\varphi} &= \frac{2hE}{1-\nu^2} \left[\frac{\nu}{2} \left(\frac{\pi}{l}\right)^2 f^2 \sin^2 \frac{\pi x}{l} + \frac{f}{R} \left(\cos \frac{\pi x}{l} + 1 \right) \right] - \frac{2hES}{3(1-\nu)} \\ M_{xx} &= \frac{2h^3E}{3(1-\nu^2)} \left[\left(\frac{\pi}{l}\right)^2 f \cos \frac{\pi x}{l} \right] \\ M_{\varphi\varphi} &= \frac{2h^3\nu E}{3(1-\nu^2)} \left[\left(\frac{\pi}{l}\right)^2 f \cos \frac{\pi x}{l} \right] \end{aligned}$$

Let us take the following notation:

$$N_1 = \frac{2hE}{1-\nu^2} \frac{1}{2} \left(\frac{\pi}{l}\right)^2 f^2; \quad N_2 = \frac{2hE}{1-\nu^2} \frac{f}{R}; \quad N_0 = \frac{2hE}{1-\nu^2} - \frac{2hES}{3(1-\nu)}; \quad M = \frac{2h^3E}{3(1-\nu^2)} \left(\frac{\pi}{l}\right)^2 f$$

Taking these designations into account, the approximations of forces and moments will take the following form:

$$\begin{cases} N_{xx} = N_1 \sin^2 \frac{\pi x}{l} + v N_2 \cos \frac{\pi x}{l} + N_0 \\ N_{\varphi\varphi} = v N_1 \sin^2 \frac{\pi x}{l} + N_2 \cos \frac{\pi x}{l} + N_0 \\ M_{xx} = M \cos \frac{\pi x}{l}; M_{\varphi\varphi} = v M \cos \frac{\pi x}{l} \end{cases} \quad (10)$$

The selected coordinate system is orthogonal. It should be noted that in the orthogonal coordinate system, the covariant, contravariant, and mixed components of the tensor coincide. This fact and (1), (2), (7) given their equalities in (7), a variation of the functor will look like this:

$$\begin{aligned} \delta \mathcal{J} = \int_{-l}^l \left\{ \dot{N}_{xx} \delta \dot{\varepsilon}_{xx} + \dot{N}_{\varphi\varphi} \delta \dot{\varepsilon}_{\varphi\varphi} + \dot{M}_{xx} \delta \dot{\varkappa}_{xx} + N_{xx} \frac{\partial \dot{w}}{\partial x} \delta \frac{\partial \dot{w}}{\partial x} + \frac{d}{dT} \left[\varepsilon_{xx} - \frac{p_0}{4h^2 E} (N_{xx} - v N_{\varphi\varphi}) - \right. \right. \\ \left. \left. \frac{3p_1}{4h^4 E} (M_{xx} - v M_{\varphi\varphi}) \right] \delta \dot{N}_{xx} + \frac{d}{dT} \left[\varepsilon_{\varphi\varphi} - \frac{p_0}{4h^2 E} (N_{\varphi\varphi} - v N_{xx}) \right] \delta \dot{N}_{\varphi\varphi} + \frac{d}{dT} \left[\varkappa_{xx} - \frac{3p_1}{4h^4 E} (N_{xx} - \right. \right. \\ \left. \left. v N_{\varphi\varphi}) \right] \delta \dot{M}_{xx} - (\dot{S}_1 \delta \dot{N}_{xx} + \dot{S}_1 \delta \dot{N}_{\varphi\varphi}) \right\} dx. \end{aligned} \quad (11)$$

We take into account the following in (1): $u_x = u_\varphi = 0$; $b_{11} = b_{12} = b_{21} = 0$; $Q_1 = Q_2 = 0$; $\psi_1 = \psi_2 = \psi_3 = 0$; $\varkappa_{\varphi\varphi} = 0$; $v \partial E = \text{const}$.

$p_1 = 0$; $q_0 = q_1 = q = 0$; $C_2 = 0$; $S_2 = 0$ If we write (9) and (10) instead of (11), expand the integrals and make the coefficients of arbitrary variations δf , δN_1 , δN_2 , δM equal to zero. We obtain the following system of equations:

$$\begin{aligned} \left[\frac{f \pi^2}{8l} (2N_0 + 3N_1 + N_2) \right]' - \frac{l}{R} \left(2\dot{l}_0 + \dot{l}_1 + \dot{l}_2 \right) \frac{\pi^2}{l} \dot{f} - \frac{3\pi^2 \dot{f}}{16l^2} - \frac{\dot{f}}{4R} - \frac{\dot{f}}{16hE} (2N_0 + 3\dot{l}_1 + \dot{l}_2) \frac{\pi^2}{2} - 0 \\ \frac{3\pi^2 \dot{f}}{16l^2} - \frac{\dot{f}}{4R} - \frac{\dot{f}}{16hE} (2N_0 + \dot{l}_1 + \dot{l}_2) \frac{\pi^2}{2} - 0 \\ \frac{\pi^2 \dot{f}}{2l^2} + \frac{\dot{f}}{4h^3 E} = 0 \end{aligned} \quad (11)$$

Let us take the following dimensionless quantities:

$$\bar{f} = \frac{f}{l}; \bar{\eta} = \frac{h}{2l}; \bar{\lambda} = \frac{h}{R}; \bar{N}_1 = \frac{N_1}{hE}; \bar{N}_2 = \frac{N_2}{hE}; \bar{N}_0 = \frac{N_0}{hE}; \bar{M} = M / (h^2 E); \quad (12)$$

Considering (12) in (11):

$$\left\{ \begin{array}{l} \pi^2 (2\bar{N}_0 + 3\bar{N}_1 + \bar{N}_2) \dot{\bar{f}} + (3\pi^2 \bar{f} - \frac{\lambda}{\eta}) \dot{\bar{N}}_1 + \left(\pi^2 \bar{f} - \frac{\lambda}{\eta} \right) \dot{\bar{N}}_2 - 8\pi^2 \eta \dot{\bar{M}} = c_1 \\ \left(3\pi^2 \bar{f} - \frac{\lambda}{\eta} \right) 4\dot{\bar{f}} - 9\dot{\bar{N}}_1 - 3\dot{\bar{N}}_2 = c_2 \\ (\pi^2 \bar{f} - 3\frac{\lambda}{\eta}) 4\dot{\bar{f}} - 3\dot{\bar{N}}_1 - 9\dot{\bar{N}}_2 = c_3 \\ 16\pi^2 \eta \dot{\bar{f}} + 18\dot{\bar{M}} = 0 \end{array} \right. \quad (13)$$

Here

$$c_1 = 2\dot{\bar{N}}_0 \left(\frac{\lambda}{\eta} - \pi^2 \bar{f} \right); \quad c_2 = c_3 = 6\dot{\bar{N}}_0. \quad (14)$$

(13) the system is solved using a computer.

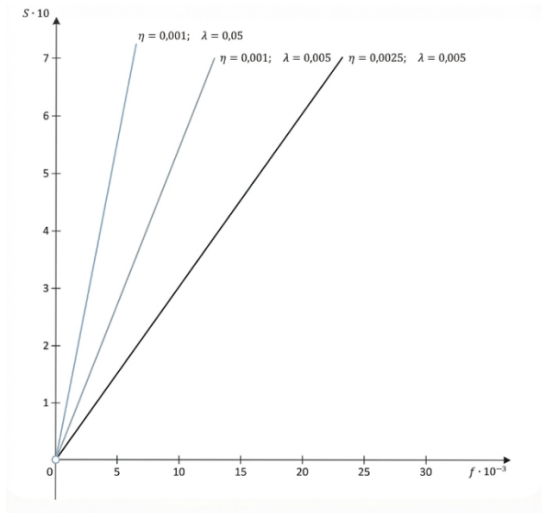


Figure 1. Dependence of the $\bar{S}$ – volume change on the dimensionless $\bar{f}$ – bend for SW 316 steel at different values of η λ and ($\bar{f}_0 = 0$).

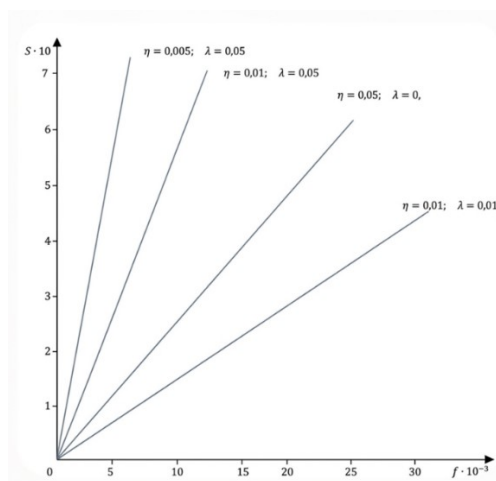


Figure 2. Dependence of the $\bar{S}$ – volume change on the dimensionless $\bar{f}$ – bend for SW 316 steel at different values of η λ and ($\bar{f}_0 = 0$).

Как видно из рисунков, при увеличении λ скорость увеличения изменения объема увеличивается, а при увеличении η скорость роста уменьшается.

RESULTS

1. Approximations were chosen that satisfy the boundary conditions of displacement, while a thin-walled cylinder made of porous material, rigidly fixed at the ends, and loses stability due to thermal expansion.
2. For a two-dimensional functional, the Euler equations were obtained in the form of a system of linear differential equations for the problem under consideration and solved numerically by the fourth-order Runge-Kutta method.
3. It is proved that with an increase in the ratio of λ - thickness to radius, the rate of increase in volume change increases, and with an increase in n - thickness to length, the growth rate decreases.

LITERATURE

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