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STRESS–STRAIN STATE AND LOAD-BEARING CAPACITY OF COMPRESSED REINFORCED CONCRETE ELEMENTS CONSIDERING CONCRETE CREEP

M.A. Hajiyev, Hajiyeva U.M., Polukhov N.Sh., Mammadov A.I, Dun Fansia
hajiyevmukhlis60@gmail.com

Abstract: As is well known, nonlinear creep of concrete having a significant effect on the stress–strain state and load-bearing capacity of reinforced concrete elements. This influence is particularly pronounced in compressed reinforced concrete members, since in such elements, in addition to long-term processes occurring in the concrete, the eccentricity of the applied compressive force and the slenderness of the member play an important role. Long-term processes in concrete lead to a redistribution of internal stresses between components and result in a considerable increase in the maximum deflection of the compressed element. It should be noted that the redistribution of internal stresses and forces is not uniform across the different components. The nonlinear creep equation used in this study accounts for the nonlinearity of both instantaneous and time-dependent deformations. Therefore, the initial parameters of the stress–strain state are also determined through a nonlinear analysis. The paper presents a differential form of the nonlinear hereditary creep equation for concrete. In solving stress relaxation problems, it is shown that the application of the fourth-order Runge–Kutta method enables the determination of stress–strain parameters with any prescribed level of accuracy.
The solution accounting for creep is reduced to solving a system of nonlinear ordinary differential equations. Corresponding software modules have been developed for both short-term static loading and long-term loading conditions. The study includes numerical results and their analysis, as well as an investigation of the influence of creep on various parameters.

***Keywords**: concrete, reinforcement, load-bearing capacity, nonlinear hereditary creep of concrete.*

Introduction. The modern theory of reinforced concrete design, both under short-term static loading and with consideration of long-term processes occurring in concrete, is being developed using a nonlinear deformation model [1,2,3, Hajiyev M., 4]. The application of the nonlinear deformation model makes it possible to determine, with high reliability, the parameters of the stress–strain state for any level of loading [1,2,3,17]. Long-term processes occurring in concrete have a significant impact on both the stress–strain state and the load-bearing capacity of reinforced concrete elements individually, as well as on structures as a whole [1,2,3,5,6,17]. Various creep theories are used in the analysis of reinforced concrete structures and their components. Among these, the nonlinear hereditary theory of concrete creep provides the most accurate description of long-term behavior [1,2,3,10,13,17]. It should be noted that recent research efforts have also been directed toward improving the creep equations themselves [7,8]. Extensive studies in this field have been carried out by Professor R.S. Sanzharovsky, who has analyzed existing creep theories and identified inaccuracies in the treatment of concrete creep within certain design standards [8,9,10]. Engineering methods for nonlinear analysis of reinforced concrete were developed by Academician V.M. Bondarenko [17]. In his work, concrete nonlinearity was incorporated for both short-term and long-term loading conditions, and separate nonlinearity functions were introduced for each case. His theory made it possible to account for time-dependent variations in both the modulus of elasticity and the strength of concrete. An applied theory of concrete creep was further developed in [6]. It should be emphasized that modern creep theories for concrete are typically expressed as nonlinear integral relationships between strain and stress. The kernels of all currently known integral equations are degenerate, and therefore, in many practical problems, differential analogs of these equations are used instead of their integral forms. In some cases, discrete models of integral creep equations are also applied [1]. Concrete creep is a complex physical phenomenon that cannot be fully described by a single equation. Consequently, research aimed at improving creep models continues to this day [8,9,11,15]. Various numerical algorithms are employed to solve creep equations [12,14,15]. Research indicates that, for sufficiently mature concrete, the time-dependent variation of the instantaneous modulus of elasticity and strength can often be neglected in creep analysis [6]. However, in reinforced concrete structures, long-term processes lead to a redistribution of the initial stress state within structural elements.

These factors have a significant influence on the load-bearing capacity of reinforced concrete elements under loading. Therefore, the study of the stress–strain state and load-bearing capacity of compressed reinforced concrete elements, taking into account creep, is a relevant and important problem. This is because the load-bearing capacity of such elements is governed both by long-term processes occurring in concrete and by the eccentricity of the applied compressive force, as well as by the slenderness of the compressed member.

1. Differential Form of the Integral Equation of Nonlinear Hereditary Concrete Creep

According to studies by the well-known specialist in the nonlinear theory of reinforced concrete, V.M. Bondarenko [17], taking into account the nonlinearity of the instantaneous elastic deformations of concrete, the equation of nonlinear concrete creep can be written in the following integral form:

$$\varepsilon_b(t) = \frac{\sigma_b(t)}{E_b(t)} \cdot S_1(\sigma_b(t)) - \int_{t_0}^{t} \sigma_b(\tau) \cdot S_2(\sigma_b(\tau)) \cdot K(t,\tau) d\tau \tag{1}$$

Here, according to the studies of V.M. Bondarenko [17], the nonlinearity functions of instantaneous and long-term deformations of concrete can be represented in the following form:

$$S_1(\sigma_b(\tau)) = 1 + \eta_1 \cdot \left(\frac{\sigma_b(\tau)}{R_b} \right)^{m_1} ; \quad S_2(\sigma_b(\tau)) = 1 + \eta_2 \cdot \left(\frac{\sigma_b(\tau)}{R_b} \right)^{m_2} \tag{2}$$

The kernel of the integral equation, taking into account the variation of the initial modulus of deformation of concrete, according to the studies of S.V. Alexandrovsky [6,11,17], has the following form:

$$K(t,\tau) = \frac{\partial}{\partial \tau} \left[\frac{1}{E_b(\tau)} - \frac{1}{E_b(t)} + C(t,\tau) \right] = -\frac{\dot{E}_b(\tau)}{E_b^2(\tau)} + \frac{\partial C(t,\tau)}{\partial \tau}$$

For sufficiently mature concrete, the creep compliance function is usually taken in the following form [6]:

$$C(t,\tau) = C_0 \cdot \left(1 - B \cdot e^{-\gamma(t-\tau)}\right) \tag{3}$$

Then, the fundamental equation of concrete creep can be written in the following form:

$$\varepsilon_b(t) = \frac{\sigma_b(t)}{E_b(t)} \cdot S_1(\sigma_b(t)) + \int_{t_0}^{t} \sigma_b(\tau) \cdot S_2(\sigma_b(\tau)) \cdot \left[\frac{\dot{E}_b(\tau)}{E_b^2(\tau)} + C_0 \cdot B \cdot \gamma \cdot e^{-\gamma \cdot (t-\tau)} \right] d\tau \tag{4}$$

Let us introduce the following notation for the integral terms:

$$X_1(t) = C_0 \cdot B \cdot \gamma \cdot e^{-\gamma \cdot t} \cdot \int_{t_0}^{t} \sigma_b(\tau) \cdot S_2(\sigma_b(\tau)) \cdot e^{\gamma \cdot \tau} d\tau \tag{5}$$

$$X_2(t) = \int_{t_0}^{t} \sigma_b(\tau) \cdot S_2(\sigma_b(\tau)) \cdot \frac{\dot{E}_b(\tau)}{E_b^2(\tau)} d\tau \tag{6}$$

hen, the main integral equation of concrete creep, in a more simplified form, can be written as follows:

$$\varepsilon_b(t) = \frac{\sigma_b(t)}{E_b} \cdot S_1(\sigma_b(t)) + X_1(t) + X_2(t) \tag{7}$$

Then, for the first derivatives of the newly introduced functions, we obtain:

$$\dot{X}(t) = -\gamma \cdot X(t) + C_0 \cdot B \cdot \gamma \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)) \tag{8}$$

$$\dot{X}_2(t) = \sigma_b(t) \cdot S_2(\sigma_b(t)) \cdot \frac{\dot{E}_b(t)}{E_b^2(t)} \tag{9}$$

Taking this into account, let us differentiate both sides of equation (7); then we obtain:

$$\begin{aligned} \dot{\varepsilon}_b(t) = & -\frac{\dot{E}_b(t)}{E_b^2(t)} \cdot \sigma_b(t) \cdot S_1(\sigma_b(t)) + \frac{\dot{\sigma}_b(t)}{E_b(t)} \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] - \\ & -\gamma \cdot X_1(t) + \left[C_0 \cdot B \cdot \gamma + \frac{\dot{E}_b(t)}{E_b^2(t)} \right] \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)) \end{aligned} \tag{10}$$

Let us differentiate both sides of equation (10) with respect to time; then we obtain:

$$\begin{aligned} \ddot{\varepsilon}_b(t) = & \left[-\frac{\ddot{E}_b(t)}{E_b^2(t)} + \frac{2\dot{E}_b^2(t)}{E_b^3(t)} \right] \cdot \sigma_b(t) \cdot S_1(\sigma_b(t)) - \frac{\dot{E}_b(t)}{E_b^2(t)} \cdot \dot{\sigma}_b(t) \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] + \\ & -\frac{\dot{E}_b(t)}{E_b^2(t)} \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] \cdot \dot{\sigma}_b(t) + \frac{\ddot{\sigma}_b(t)}{E_b} \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] + \\ & \frac{\dot{\sigma}_b^2(t)}{E_b} \cdot [2S_1'(\sigma_b(t)) + \sigma_b(t) \cdot S_1''(\sigma_b(t))] + \left[\frac{\ddot{E}_b(t)}{E_b^2(t)} - 2 \cdot \frac{\dot{E}_b^2(t)}{E_b^3(t)} \right] \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)) + \\ & + \left[C_0 \cdot B \cdot \gamma + \frac{\dot{E}_b(t)}{E_b^2(t)} \right] \cdot [S_2(\sigma_b(t)) + \sigma_b(t) \cdot S_2'(\sigma_b(t))] \cdot \dot{\sigma}_b(t) - C_0 \cdot B \cdot \gamma^2 \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)) + \gamma^2 \cdot X_1(t) \end{aligned}$$

After collecting like terms, we obtain:

$$\begin{aligned} \ddot{\varepsilon}_b(t) = & \left[-\frac{\ddot{E}_b(t)}{E_b^2(t)} + \frac{2\dot{E}_b^2(t)}{E_b^3(t)} \right] \cdot \sigma_b(t) \cdot S_1(\sigma_b(t)) - 2 \cdot \frac{\dot{E}_b(t)}{E_b^2(t)} \cdot \dot{\sigma}_b(t) \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] + \\ & + \frac{\ddot{\sigma}_b(t)}{E_b} \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] + \frac{\dot{\sigma}_b^2(t)}{E_b} \cdot [2S_1'(\sigma_b(t)) + \sigma_b(t) \cdot S_1''(\sigma_b(t))] - \\ & + \gamma^2 \cdot X_1(t) + \left[\frac{\ddot{E}_b(t)}{E_b^2(t)} - 2 \cdot \frac{\dot{E}_b^2(t)}{E_b^3(t)} - C_0 \cdot B \cdot \gamma^2 \right] \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)) + \\ & + \left[C_0 \cdot B \cdot \gamma + \frac{\dot{E}_b(t)}{E_b^2(t)} \right] \cdot [S_2(\sigma_b(t)) + \sigma_b(t) \cdot S_2'(\sigma_b(t))] \cdot \dot{\sigma}_b(t) \end{aligned} \tag{11}$$

To eliminate the integral term, let us multiply both sides of equation (10) by a certain factor and then add it term-by-term to equation (11); as a result, we obtain:

$$\begin{aligned}\ddot{\varepsilon}_b(t)+\gamma\cdot\dot{\varepsilon}_b(t)=&\frac{\ddot{\sigma}_b(t)}{E_b(t)}\cdot[S_1(\sigma_b(t))+\sigma_b(t)\cdot S_1'(\sigma_b(t))]+\frac{\dot{\sigma}_b^2(t)}{E_b(t)}\cdot[2S_1'(\sigma_b(t))+\sigma_b(t)\cdot S_1''(\sigma_b(t))]+\\&+\left[\frac{\gamma}{E_b(t)}-2\cdot\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot[S_1(\sigma_b(t))+\sigma_b(t)\cdot S_1'(\sigma_b(t))]\cdot\dot{\sigma}_b(t)+\\&+\left[-\frac{\ddot{E}_b(t)}{E_b^2(t)}+\frac{2\dot{E}_b^2(t)}{E_b^3(t)}-\gamma\cdot\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\sigma_b(t)\cdot S_1(\sigma_b(t))+\\&+\left[C_0\cdot B\cdot\gamma+\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot[S_2(\sigma_b(t))+\sigma_b(t)\cdot S_2'(\sigma_b(t))]\cdot\dot{\sigma}_b(t)+\\&+\left[\frac{\ddot{E}_b(t)}{E_b^2(t)}-\frac{2\dot{E}_b^2(t)}{E_b^3(t)}+\gamma\cdot\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\sigma_b(t)\cdot S_2(\sigma_b(t))\end{aligned}\qquad(12)$$

Thus, the integral equation of nonlinear hereditary creep of concrete is represented in an equivalent differential form. This differential equation must satisfy the following initial conditions, which follow from equations (4) and (10):

$$\varepsilon_b(t_0)=\frac{\sigma_b(t_0)}{E_b(t_0)}\cdot S_1(\sigma_b(t_0))\qquad(13)$$

$$\begin{aligned}\dot{\varepsilon}_b(t_0)=&-\frac{\dot{E}_b(t_0)}{E_b^2(t_0)}\cdot\sigma_b(t_0)\cdot S_1(\sigma_b(t_0))+\frac{\dot{\sigma}_b(t_0)}{E_b(t_0)}\cdot[S_1(\sigma_b(t_0))+\sigma_b(t)\cdot S_1'(\sigma_b(t_0))]+\\&+\left[C_0\cdot B\cdot\gamma+\frac{\dot{E}_b(t_0)}{E_b^2(t_0)}\right]\cdot\sigma_b(t_0)\cdot S_2(\sigma_b(t_0))\end{aligned}\qquad(14)$$

For the corresponding derivatives of the nonlinearity functions of instantaneous elastic and time-dependent deformations, the following expressions can be written:

$$S_i'(\sigma_b(\tau))=\frac{\eta_i\cdot m_i}{R_b}\cdot\left(\frac{\sigma_b(\tau)}{R_b}\right)^{m_i-1};\qquad S_i''(\sigma_b(\tau))=\frac{\eta_i\cdot m_i\cdot(m_i-1)}{R_b^2}\cdot\left(\frac{\sigma_b(\tau)}{R_b}\right)^{m_i-2};\qquad i=1,2\qquad(15)$$

It is clear that for mature concrete, when the increase in the initial modulus of elasticity can be neglected i.e., when $E_b(t)=E_b=const$, the relationships obtained above are significantly simplified and take the following form:

$$\varepsilon_b(t_0)=\frac{\sigma_b(t_0)}{E_b}\cdot S_1(\sigma_b(t_0))\qquad(16)$$

$$\dot{\varepsilon}_b(t)+\gamma\cdot\varepsilon_b(t)=\frac{\dot{\sigma}_b(t)}{E_b}\cdot[S_1(\sigma_b(t))+\sigma_b(t)\cdot S_1'(\sigma_b(t))]+\frac{\gamma}{E_b}\cdot\sigma_b(t)\cdot S_1(\sigma_b(t))+C_0\cdot B\cdot\gamma\cdot\sigma_b(t)\cdot S_2(\sigma_b(t))\qquad(17)$$

2. Solution of Relaxation Problems.

For a given value of the initial strain $\varepsilon_b(t_0)$ the initial stress $\sigma_b(t_0)$ is determined from the solution of the nonlinear equation (12). After that, with the known value of the initial stress, the initial rate of stress is found as the solution of the linear equation (13). Assuming $\dot{\sigma}_b(t)=\sigma_{b1}(t)$ the main nonlinear second-order differential equation can be written in the following standard form:

$$\begin{aligned}\dot{\sigma}_b(t)&=\sigma_{b1}(t)\\\dot{\sigma}_{b1}(t)&=\Omega(t,\sigma_b(t),\sigma_{b1}(t))\end{aligned}\qquad(18)$$

Here, the following notation is introduced:

$$\Omega(t,\sigma_b(t),\sigma_{b1}(t))=\frac{E_b(t)}{S_1(\sigma_b(t))+\sigma_b(t)\cdot S_1'(\sigma_b(t))}\cdot\{\ddot{\varepsilon}_b(t)+\gamma\cdot\dot{\varepsilon}_b(t)-\psi(\sigma_b(t),\sigma_{b1}(t))\},$$

$$\psi(\sigma_b(t),\sigma_{b1}(t)) = \frac{\sigma_{b1}^2(t)}{E_b(t)}\cdot\left[2S_1'(\sigma_b(t)) + \sigma_b(t)\cdot S_1''(\sigma_b(t))\right] +$$
$$+\left[\frac{\gamma}{E_b(t)} - \frac{2\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\left[S_1(\sigma_b(t)) + \sigma_b(t)\cdot S_1'(\sigma_b(t))\right]\cdot\sigma_{b1}(t) +$$
$$+\left[-\frac{\ddot{E}_b(t)}{E_b^2(t)} + 2\cdot\frac{\dot{E}_b^2(t)}{E_b^3(t)} - \gamma\cdot\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\sigma_b(t)\cdot S_1(\sigma_b(t)) + \tag{19}$$
$$+\left[C_0\cdot B\cdot\gamma + \frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\left[S_2(\sigma_b(t)) + \sigma_b(t)\cdot S_2'(\sigma_b(t))\right]\cdot\sigma_{b1}(t) +$$
$$+\left[\frac{\ddot{E}_b(t)}{E_b^2(t)} - 2\cdot\frac{\dot{E}_b^2(t)}{E_b^3(t)} + \gamma\cdot\frac{\dot{E}_b(t)}{E_b^2(t)}\right]\cdot\sigma_b(t)\cdot S_2(\sigma_b(t))$$

The obtained normal system of differential equations (17) is solved numerically using the fourth-order Runge–Kutta method [18]. The initial values of the stress and its derivative are determined from relations (12) and (13). Let us choose a time step Δt. Then, at the next step, the values of the unknown functions are given by:

$$\begin{aligned}\sigma_b(t_j) &= \sigma_b(t_{j-1}) + \frac{1}{6}\cdot(k_{11} + 2k_{21} + 2k_{31} + k_{41})\\ \sigma_{b1}(t_j) &= \sigma_{b1}(t_{j-1}) + \frac{1}{6}\cdot(k_{12} + 2k_{22} + 2k_{32} + k_{42})\end{aligned} \tag{20}$$

Here:

$$\begin{aligned}
&k_{11} = \Delta t\cdot\sigma_{b1}(t_{j-1}) && k_{12} = \Delta t\cdot\Omega(t_{j-1},\sigma_{b1}(t_{j-1}),\sigma_b(t_{j-1}))\\
&k_{21} = \Delta t\cdot\left(\sigma_{b1}(t_{j-1}) + \frac{k_{11}}{2}\right) && k_{22} = \Delta t\cdot\Omega\left(t_{j-1} + \frac{\Delta t}{2},\sigma_{b1}(t_{j-1}) + \frac{k_{11}}{2},\sigma_b(t_{j-1}) + \frac{k_{12}}{2}\right)\\
&k_{31} = \Delta t\cdot\left(\sigma_{b1}(t_{j-1}) + \frac{k_{21}}{2}\right) ; && k_{31} = \Delta t\cdot\Omega\left(t_{j-1} + \frac{\Delta t}{2},\sigma_{b1}(t_{j-1}) + \frac{k_{21}}{2},\sigma_b(t_{j-1}) + \frac{k_{22}}{2}\right)\\
&k_{41} = \Delta t\cdot(\sigma_{b1}(t_{j-1}) + k_{31}) && k_{41} = \Delta t\cdot\Omega(t_{j-1} + \Delta t,\sigma_{b1}(t_{j-1}) + k_{31},\sigma_b(t_{j-1}) + k_{32})
\end{aligned} \tag{21}$$

A corresponding software module for solving relaxation problems has been developed in the Turbo Pascal ABC programming language. Using this program, various numerical experiments and a parametric analysis of stress relaxation were carried out. To evaluate the accuracy of the proposed discrete formulation, the same relaxation problem was solved using different time integration steps. The results of these calculations are presented in Table 1. It was established that the proposed numerical algorithm provides sufficiently high accuracy.

For example, in the cases considered, increasing the integration step from $\Delta t = 0{,}5\ day$ to $\Delta t = 5\ days$ does not lead to any noticeable change in the final relaxed stress. Moreover, calculations performed both with and without accounting for the variation of the initial modulus of elasticity of concrete have shown that the increase in the initial modulus has only a minor effect on the final value of the relaxed stress for all concrete grades. A similar result was obtained with respect to the age of the concrete at the time of loading. The final resulting stresses for concrete ages of $t_0 = 7\ days$ and $t_0 = 28\ days$ differ by no more than 2,0%. In parallel, the influence of the nonlinearity of instantaneous elastic deformations was investigated. The calculations demonstrated that this nonlinearity has a significant effect on the magnitude of the final relaxed stress and therefore must be taken into account when considering creep. The performed calculations also showed that, for sufficiently mature concrete, the variation of the initial modulus of elasticity can be neglected within engineering accuracy when accounting for creep, and a simplified form of the nonlinear hereditary creep equations (16) and (17) may be used. Therefore, in the subsequent analysis of the stress–strain state and load-bearing capacity of compressed reinforced concrete elements, these simplified equations will be adopted.

Table 1. Values of final relaxed stresses for different concrete grades at various time integration steps.

Concrete grade	Final relaxed stress		
	Variable modulus of elasticity; nonlinear instantaneous elastic deformations	Constant modulus of elasticity; nonlinear instantaneous elastic deformations	Variable modulus of elasticity; linear instantaneous elastic deformations
	$\sigma_{b(7,\infty)}$; MPa the initial stress is taken as 50% of the compressive strength		
B20	4,9415	5,0314	4,6361
B30	7,7717	7,2256	6,7780
B40	9,2516	9,2225	8,6972
B50	11,4305	11,3701	10,7150
B60	13,7507	13,6233	12,7968
Concrete grade	Final relaxed stress		
	Variable modulus of elasticity; nonlinear instantaneous elastic deformations	Constant modulus of elasticity; nonlinear instantaneous elastic deformations	Variable modulus of elasticity; linear instantaneous elastic deformations
	$\sigma_{b(28,\infty)}$; MPa the initial stress is taken as 50% of the compressive strength		
B20	4,9612	5,0314	4,7376
B30	7,1289	7,2256	6,8324
B40	9,0792	9,2053	8,6726
B50	11,2273	11,3701	10,7017
B60	13,4633	13,6233	12,7734
Concrete grade	Final relaxed stress		
	Variable modulus of elasticity; nonlinear instantaneous elastic deformations	Constant modulus of elasticity; nonlinear instantaneous elastic deformations	Variable modulus of elasticity; linear instantaneous elastic deformations
	$\sigma_{b(7,\infty)}$; MPa the initial stress is taken as 80% of the compressive strength		
B20	8,4034	8,6109	6,2082
B30	12,1851	12,4100	9,4289
B40	15,6750	15,7912	12,3918
B50	19,0117	19,2999	15,2939
B60	22,5187	22,9016	18,2761
Concrete grade	Final relaxed stress		
	Variable modulus of elasticity; nonlinear	Constant modulus of elasticity; nonlinear	Variable modulus of elasticity; linear

	instantaneous elastic deformations	instantaneous elastic deformations	instantaneous elastic deformations
	$\sigma_{b(28,\infty)}$; *MPa* the initial stress is taken as 80% of the compressive strength		
B20	8,5215	8,6109	6,9589
B30	12,2743	12,4100	10,2559
B40	15,6381	15,7912	13,2296
B50	19,0540	19,2999	16,2819
B60	22,5877	22,9016	19,4389

3. Stress–strain state and load-bearing capacity of compressed reinforced concrete elements.

For the sake of specificity, reinforced concrete elements with a rectangular cross-section are considered in the general case with double reinforcement. Depending on the magnitude of the eccentricity of the compressive force, two possible positions of the neutral axis may occur (see Scheme 1).

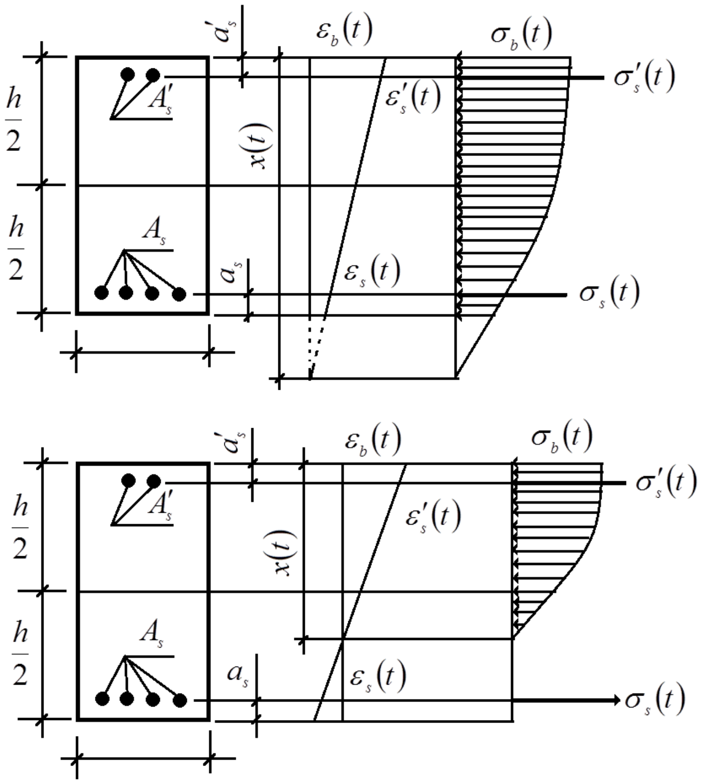


Figure 1. Design scheme of the cross-section of an eccentrically compressed element

For a composite section, according to the nonlinear deformation model of reinforced concrete, the hypothesis of plane sections is assumed to be valid. According to the studies in [17], the distribution of normal stresses over the height of the element can be taken as a power function. Therefore, the normal stresses in concrete can be expressed as follows:

$$\sigma_{bz} = \sigma_b \cdot \left(\frac{z + x - \frac{h}{2}}{x} \right)^{n_\sigma} \qquad (22)$$

Here, in accordance with [17], the stress block parameter is also expressed in terms of the stresses in the extreme compressed fiber of concrete.

$$n_\sigma = 1 - (1 - f_0) \cdot \left(\frac{\sigma_b}{R_b} \right)^{m_\sigma} \tag{23}$$

Then, for the axial force and bending moment due to compressive stresses in the concrete, we have:

$$N_b(\sigma_b, n_\sigma, x) = \int_q^{\frac{h}{2}} \sigma_{bz} \cdot b \cdot dz = \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 1} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] \tag{24}$$

$$\begin{aligned} M_b(\sigma_b, n_\sigma, x) = \int_q^{\frac{h}{2}} \sigma_{bz} \cdot z \cdot b \cdot dz = \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \right] - \\ - \frac{\sigma_b \cdot b}{n_\sigma + 1} \cdot \left(x - \frac{h}{2} \right) \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] \end{aligned} \tag{25}$$

Using the plane sections hypothesis, the strains in the reinforcement bars can be expressed in terms of the strain of the extreme compressed fiber of the concrete as follows:

$$\varepsilon'_s = \frac{\varepsilon_b}{x} \cdot (x - a'_s), \quad \varepsilon_s = \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \tag{26}$$

Then, for the stresses in the reinforcement bars with an idealized elastic–perfectly plastic behavior (with a distinct yield plateau), the following expressions can be written:

$$\sigma'_s(\varepsilon_b, x) = \begin{cases} E'_s \cdot \frac{\varepsilon_b}{x} \cdot (x - a'_s), & if \ \left| \frac{\varepsilon_b}{x} \cdot (x - a'_s) \right| \le \varepsilon_{st} \\ R'_s, \ if \ \left| \frac{\varepsilon_b}{x} \cdot (x - a'_s) \right| > \varepsilon_{st} \end{cases} \tag{27}$$

$$\sigma_s(\varepsilon_b, x) = \begin{cases} E_s \cdot \frac{\varepsilon_b}{x} \cdot (x - h + a_s), & if \ \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| \le \varepsilon_{st} \\ R_s \cdot sign(R_s), \ if \ \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| > \varepsilon_{st} \end{cases} \tag{28}$$

Taking these expressions into account, we can formulate the expressions for the axial force and bending moment in the section due to stresses in the reinforcement bars as follows:

$$N_s(\varepsilon_b, x) = \sigma'_s(\varepsilon_b, x) \cdot A'_s + \sigma_s(\varepsilon_b, x) \cdot A_s \tag{29}$$

$$M_s(\varepsilon_b, x) = \sigma'_s(\varepsilon_b, x) \cdot A'_s \cdot \left(\frac{h}{2} - a'_s \right) - \sigma_s(\varepsilon_b, x) \cdot A_s \cdot \left(\frac{h}{2} - a_s \right) \tag{30}$$

Considering the deformation of the member axis, we formulate the equilibrium equations for the most stressed cross-section.

$$N_b(\sigma_b, n_\sigma, x) + N_s(\varepsilon_b, x) = P \tag{31}$$

$$M_b(\sigma_b, n_\sigma, x) + M_s(\varepsilon_b, x) = P \cdot (e + f) \tag{32}$$

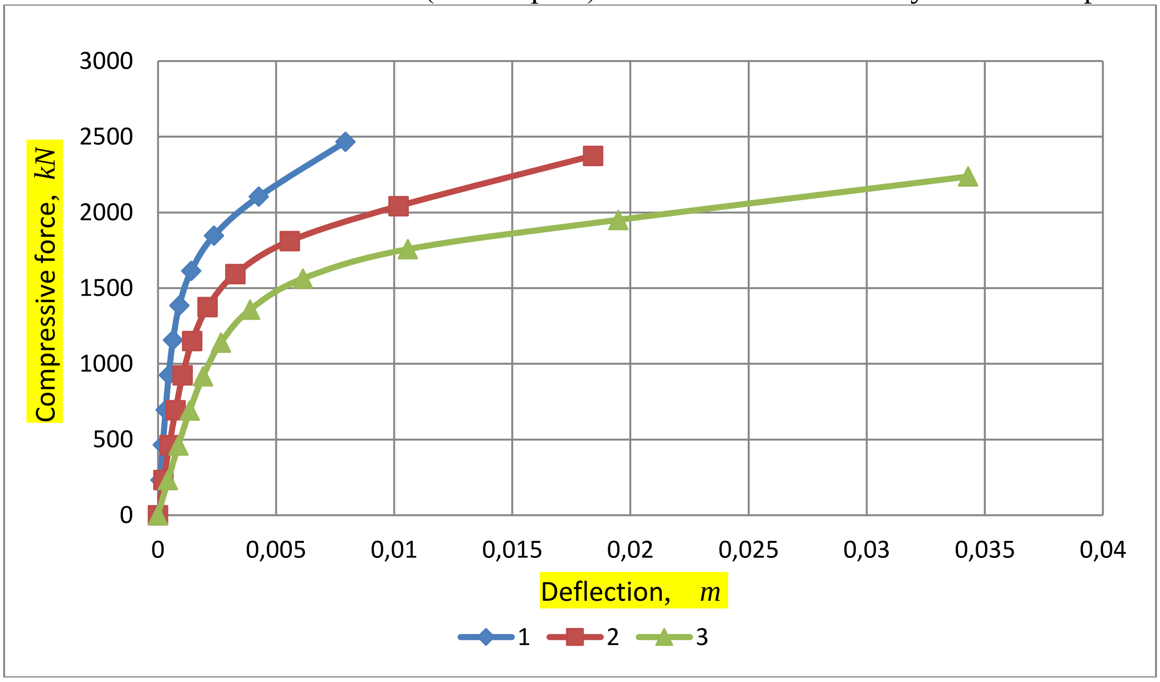
The deflected shape of the member axis can be approximated by a suitable function of the form

$w(y) = f \cdot w_0(y)$ which allows the maximum deflection of the member to be expressed in terms of the parameters ε_b and x as follows [1,2,3] (33), $f = \rho_0 \cdot \frac{\varepsilon_b}{x}$ (33), where $\rho_0 = \frac{1}{|w''(y_0)|}$. Here y_0 - is the coordinate of the most highly stressed cross-section of the member.

3.1. Determination of the Initial Conditions for the Cauchy Problem. Thus, equations (16), (23), (24), (25), (26), (27), (28), (29), (30), (31), (32), and (23) constitute the governing system of nonlinear equations for the problem under consideration at the initial moment of time, without accounting for creep. Of particular interest is the investigation of the stress–strain state and load-bearing capacity under short-term static loading. A detailed description of such an analysis is presented in the works of the first author [1,2,3]. The essence of this calculation algorithm is as follows: the value of stress $\sigma_b \in [0; R_b]$. Then, from equations (15) and (23), the corresponding values of ε_b and n_σ are determined. After that, by eliminating the load parameter P from equations (31) and (32), the following equation is obtained for determining the parameter that defines the position of the neutral axis:

$$\Psi(x) = M_b(\sigma_b, n_\sigma, x) + M_s(\varepsilon_b, x) - (M_b(\sigma_b, n_\sigma, x) + M_s(\varepsilon_b, x)) \cdot \left(e + \rho_0 \cdot \frac{\varepsilon_b}{x} \right) = 0 \qquad (34)$$

Thus, for a given value of stress, the parameters defining the stress–strain state can be determined uniquely with any prescribed level of accuracy. In particular, based on the calculation results, it is possible to establish the "load–deflection" relationship, which is then used to determine the load-bearing capacity of the compressed reinforced concrete element under short-term static loading. The described algorithm is readily programmable. A corresponding software module has been developed, and a series of numerical experiments have been carried out using it. With the application of the developed software module, an eccentrically compressed reinforced concrete element was analyzed under the following initial conditions: $b = 0{,}4\ m$; $h = 0{,}6\ m$, $e = 0{,}01\ m$, concrete $B\,20$ $R_b = 11{,}5\ MPa$; $E_b = 27 \cdot 10^3\ MPa$, $\eta_1 = 2{,}0$; $m_1 = 4{,}7$; $\eta_2 = 2{,}35$; $m_2 = 4{,}0$; $f_0 = 0{,}11$; $m_\sigma = \frac{2m_1}{3}$; rebars $A\,400$; $4\varnothing 28$; $A'_s = 0$; $A_s = 24{,}63\ sm^2$; $a_s = 4{,}4\ sm$. Under these conditions, for different values of the member length $l_0 = 6\ m$; $l_0 = 9\ m$; $l_0 = 12\ m$ i.e., for different values of the slenderness of the compressed element, "load–deflection" curves were constructed (see Graph 1) for the case of nominally central compression.

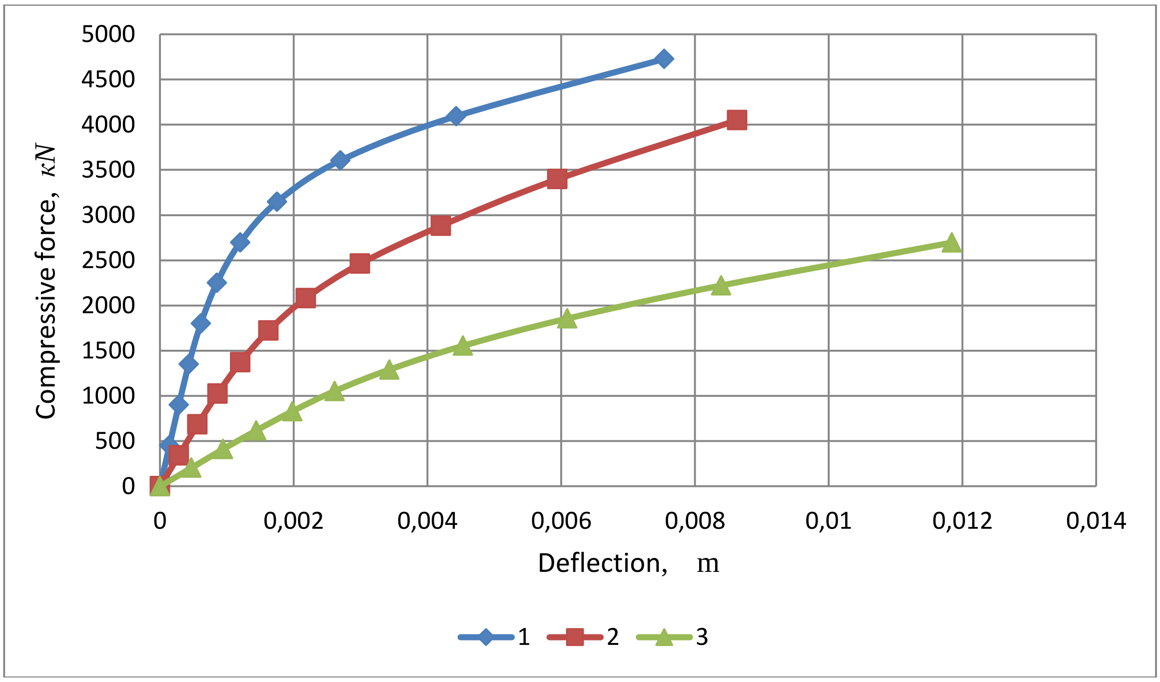


Graph 1. Load–deflection curves under nominally concentric compression, $e = 0{,}01\ m$.

1 - $l_0 = 6\ m$; 2 - $l_0 = 9\ m$; 3 - $l_0 = 12\ m$.

As can be seen from the presented curves, for this example the descending branches are not observed in the "load–deflection" diagrams. This indicates that the load-bearing capacity of the element is governed by the strength condition. In parallel, calculations were performed for a member length of

$l_0 = 6\ m$ and concrete grade $B\,40$ at various values of the eccentricity of the compressive force (see Graph 2). As follows from the results, the influence of eccentricity on the load-bearing capacity of compressed elements is significant. For example, in the case considered, at $e = 1\ sm$; $e = 5\ sm$; and $e = 15\ sm$ the corresponding values of the load-bearing capacity are $P_1 = 4725{,}8\ kN$; $P_5 = 4050{,}6\ kN$; $P_{15} = 2696{,}7\ kN$; $f_1 = 7{,}54\ mm$; $f_5 = 8{,}63\ mm$; $f_{15} = 11{,}84\ mm$. It can be observed that with an increase in eccentricity from $e = 1\ sm$ to $e = 5\ sm$; and $e = 15\ sm$ the load-bearing capacity decreases, compared to nominally concentric compression, by 14.29% and 42.94%, respectively. At the same time, the maximum deflection increases by 14.46% and 57.03%, respectively, compared to nominally concentric compression. This example clearly demonstrates that, in compressed reinforced concrete elements, both the load-bearing capacity and the stress–strain state depend on many factors, even without considering concrete creep. Therefore, the analysis of such elements should be carried out using models that adequately reflect their behavior under loading.



Graph 2. Load–deflection curves: 1 - $e = 1\ sm$; 2 - $e = 5\ sm$; 3 - $e = 15\ sm$.

3.2. Analysis of the Stress–Strain State Considering Nonlinear Hereditary Concrete Creep. As shown above, the principal governing relationships of the problem under consideration are given by the following expressions:

$$\dot{\varepsilon}_b(t) + \gamma \cdot \varepsilon_b(t) = \frac{\dot{\sigma}_b(t)}{E_b} \cdot [S_1(\sigma_b(t)) + \sigma_b(t) \cdot S_1'(\sigma_b(t))] + \frac{\gamma}{E_b} \cdot \sigma_b(t) \cdot S_1(\sigma_b(t)) + C_0 \cdot B \cdot \gamma \cdot \sigma_b(t) \cdot S_2(\sigma_b(t)),$$

$$n_\sigma = 1 - (1 - f_0) \cdot \left(\frac{\sigma_b}{R_b}\right)^{m_\sigma}$$

$$N_b = \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 1} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right],$$

$$M_b = \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \right] - \frac{\sigma_b \cdot b}{n_\sigma + 1} \cdot \left(x - \frac{h}{2} \right) \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right],$$

$$\sigma_s = \begin{cases} E_s \cdot \frac{\varepsilon_b}{x} \cdot (x - h + a_s), & if \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| \le \varepsilon_{st} \\ R_s \cdot sign(R_s), & if \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| > \varepsilon_{st} \end{cases},$$

$$\sigma'_s(\varepsilon_b, x) = \begin{cases} E'_s \cdot \frac{\varepsilon_b}{x} \cdot (x - a'_s), & if \left| \frac{\varepsilon_b}{x} \cdot (x - a'_s) \right| \le \varepsilon_{st} \\ R'_s, & if \left| \frac{\varepsilon_b}{x} \cdot (x - a'_s) \right| > \varepsilon_{st} \end{cases}$$

$$N_s(\varepsilon_b, x) = \sigma'_s(\varepsilon_b, x) \cdot A'_s + \sigma_s(\varepsilon_b, x) \cdot A_s,$$

$$M_s = \sigma'_s(\varepsilon_b, x) \cdot A'_s \cdot \left(\frac{h}{2} - a'_s \right) - \sigma_s(\varepsilon_b, x) \cdot A_s \cdot \left(\frac{h}{2} - a_s \right),$$

$$N_b(\sigma_b, n_\sigma, x) + N_s(\varepsilon_b, x) = P,$$

$$M_b(\sigma_b, n_\sigma, x) + M_s(\varepsilon_b, x) = P \cdot (e + f),$$

$$f = \rho_0 \cdot \frac{\varepsilon_b}{x}.$$

The resulting set consists of 11 equations with respect to the unknown parameters $\varepsilon_b, \sigma_b, N_b, M_b, x, f, \sigma_s, \sigma'_s, N_s, M_s, n_\sigma$. Since the fundamental equation of concrete creep is a first-order differential equation, let us differentiate both the left-hand and right-hand sides of the above relationships with respect to time once; then we obtain:

$$\dot{n}_\sigma = A_9(\sigma_b) \cdot \dot{\sigma}_b, \tag{35}$$

$$\dot{N}_b = A_1(\sigma_b, x, n_\sigma) \cdot \dot{\sigma}_b + A_2(\sigma_b, x, n_\sigma) \cdot \dot{n}_\sigma + A_3(\sigma_b, x, n_\sigma) \cdot \dot{x} \tag{36}$$

$$\dot{M}_b = A_4(\sigma_b, x, n_\sigma) \cdot \dot{\sigma}_b + A_5(\sigma_b, x, n_\sigma) \cdot \dot{n}_\sigma + A_6(\sigma_b, x, n_\sigma) \cdot \dot{x} \tag{37}$$

$$\dot{\sigma}_s = A_7(\varepsilon_b, x) \cdot \dot{\varepsilon}_b + A_8(\varepsilon_b, x) \cdot \dot{x} \tag{38}$$

$$\dot{N}_s = A_s \cdot \dot{\sigma}_s + A'_s \cdot \dot{\sigma}'_s \tag{39}$$

$$\dot{M}_s = -A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot \dot{\sigma}_s + A'_s \cdot \left(\frac{h}{2} - a'_s \right) \cdot \dot{\sigma}'_s \tag{40}$$

$$A_1(\sigma_b, x, n_\sigma) \cdot \dot{\sigma}_b + A_2(\sigma_b, x, n_\sigma) \cdot \dot{n}_\sigma + A_3(\sigma_b, x, n_\sigma) \cdot \dot{x} + A_s \cdot \dot{\sigma}_s + A'_s \cdot \dot{\sigma}'_s = \dot{P} \tag{41}$$

$$\begin{aligned} A_4(\sigma_b, x, n_\sigma) \cdot \dot{\sigma}_b + A_5(\sigma_b, x, n_\sigma) \cdot \dot{n}_\sigma + A_6(\sigma_b, x, n_\sigma) \cdot \dot{x} - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot \dot{\sigma}_s + A'_s \cdot \left(\frac{h}{2} - a'_s \right) \cdot \dot{\sigma}'_s = \\ = \dot{P} \cdot \left(e + \rho_0 \cdot \frac{\varepsilon_b}{x} \right) + P \cdot \dot{f} \end{aligned} \tag{42}$$

$$\dot{f} = \frac{\rho_0}{x} \cdot \dot{\varepsilon}_b - \rho_0 \cdot \frac{\varepsilon_b}{x^2} \cdot \dot{x} \tag{43}$$

$$\dot{\sigma}'_s = A_{12}(\varepsilon_b, x) \cdot \dot{\varepsilon}_b + A_{13}(\varepsilon_b, x) \cdot \dot{x} \tag{44}$$

In the above expressions, the following notations are introduced:

$$A_1(\sigma_b, x, n_\sigma) = \frac{b \cdot x}{n_\sigma + 1} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right]$$

$$A_2(\sigma_b, x, n_\sigma) = -\frac{\sigma_b \cdot b \cdot x}{(n_\sigma + 1)^2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] - \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 1} \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \ln \left(\frac{q + x - \frac{h}{2}}{x} \right)$$

$$A_3(\sigma_b, x, n_\sigma) = \frac{\sigma_b \cdot b}{n_\sigma + 1} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] - \frac{\sigma_b \cdot b}{x} \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma} \cdot \left(q' - q + \frac{h}{2} \right)$$

$$A_4(\sigma_b, x, n_\sigma) = \frac{b \cdot x}{n_\sigma + 2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \right] - \frac{b}{n_\sigma + 1} \cdot \left(x - \frac{h}{2} \right) \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right]$$

$$A_5(\sigma_b, x, n_\sigma) = -\frac{\sigma_b \cdot b \cdot x}{(n_\sigma + 2)^2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \right] - \frac{\sigma_b \cdot b \cdot x}{n_\sigma + 2} \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \cdot \ln \left(\frac{q + x - \frac{h}{2}}{x} \right) +$$

$$+ \frac{\sigma_b \cdot b}{(n_\sigma + 1)^2} \cdot \left(x - \frac{h}{2} \right) \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] + \frac{\sigma_b \cdot b}{n_\sigma + 1} \cdot \left(x - \frac{h}{2} \right) \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \cdot \ln \left(\frac{q + x - \frac{h}{2}}{x} \right)$$

$$A_6(\sigma_b, x, n_\sigma) = \frac{\sigma_b \cdot b}{n_\sigma + 2} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 2} \right] - \frac{\sigma_b \cdot b}{x} \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \cdot \left(q' - q + \frac{h}{2} \right) -$$

$$- \frac{\sigma_b \cdot b}{n_\sigma + 1} \cdot \left[1 - \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma + 1} \right] + \frac{\sigma_b \cdot b}{x^2} \cdot \left(x - \frac{h}{2} \right) \cdot \left(\frac{q + x - \frac{h}{2}}{x} \right)^{n_\sigma} \cdot \left(q' - q + \frac{h}{2} \right)$$

$$A_7(\varepsilon_b, x) = \begin{cases} E_s \cdot \frac{1}{x} \cdot (x - h + a_s), & if \quad \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| \le \varepsilon_{st} \\ 0\,, \ if \quad \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| > \varepsilon_{st} \end{cases}$$

$$A_8(\varepsilon_b, x) = \begin{cases} E_s \cdot \varepsilon_b \cdot \frac{h - a_s}{x^2}, & if \quad \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| \le \varepsilon_{st} \\ 0\,, \ if \quad \left| \frac{\varepsilon_b}{x} \cdot (x - h + a_s) \right| > \varepsilon_{st} \end{cases}$$

$$A_9(\sigma_b) = -(1 - f_0) \cdot m_\sigma \cdot \left(\frac{\sigma_b}{R_b} \right)^{m_\sigma - 1} \cdot \frac{1}{R_b}$$

$$A_{12}(\varepsilon_b, x) = \begin{cases} E_s' \cdot \dfrac{1}{x} \cdot (x - a_s), & if \quad \left| \dfrac{\varepsilon_b}{x} \cdot (x - a_s) \right| \le \varepsilon_{st} \\ 0, \ if & \left| \dfrac{\varepsilon_b}{x} \cdot (x - a_s) \right| > \varepsilon_{st} \end{cases}$$

$$A_{13}(\varepsilon_b, x) = \begin{cases} E_s \cdot \varepsilon_b \cdot \dfrac{a_s}{x^2}, & if \quad \left| \dfrac{\varepsilon_b}{x} \cdot (x - ha_s) \right| \le \varepsilon_{st} \\ 0, \ if & \left| \dfrac{\varepsilon_b}{x} \cdot (x - a_s) \right| > \varepsilon_{st} \end{cases} \tag{45}$$

Introducing the following notation:

$$A_{10}(\sigma_b) = \frac{1}{E_b} \cdot [S_1(\sigma_b) + \sigma_b \cdot S_1'(\sigma_b)]; \quad A_{11}(\sigma_b, \varepsilon_b) = \frac{\gamma}{E_b} \cdot \sigma_b \cdot S_1(\sigma_b) + C_0 \cdot B \cdot \gamma \cdot \sigma_b \cdot S_2(\sigma_b) - \gamma \cdot \varepsilon_b \tag{46}$$

Introducing the following notation, the main differential equation of creep (17) can be written in a more compact form as follows:

$$\dot{\varepsilon}_b = \dot{\sigma}_b(t) \cdot A_{10}(\sigma_b) + A_{11}(\sigma_b, \varepsilon_b) \tag{47}$$

In the above relations (38), (42), and (43), the corresponding derivatives can be expressed in terms of the derivatives $\dot{\sigma}_b$ and $\dot{x}$:

$$\dot{\sigma}_s = A_7(\sigma_b, x) \cdot A_{10}(\sigma_b) \cdot \dot{\sigma}_b(t) + A_8(\sigma_b, x) \cdot \dot{x} + A_7(\sigma_b, x) \cdot A_{11}(\sigma_b, \varepsilon_b) \tag{48}$$

$$\dot{\sigma}_s' = A_{12}(\sigma_b, x) \cdot A_{10}(\sigma_b) \cdot \dot{\sigma}_b(t) + A_{13}(\sigma_b, x) \cdot \dot{x} + A_{12}(\sigma_b, x) \cdot A_{11}(\sigma_b, \varepsilon_b) \tag{49}$$

$$\dot{f} = A_{10}(\sigma_b) \cdot \frac{\rho_0}{x} \cdot \dot{\sigma}_b - \rho_0 \cdot \frac{\varepsilon_b}{x^2} \cdot \dot{x} + A_{11}(\sigma_b, \varepsilon_b) \cdot \frac{\rho_0}{x} \tag{50}$$

Taking into account relations (48), (49), and (50), let us simplify equations (36) and (37); then we obtain:

$$\dot{N}_b = [A_1(\sigma_b, x, n_\sigma) + A_2(\sigma_b, x, n_\sigma) \cdot A_9(\sigma_b)] \cdot \dot{\sigma}_b + A_3(\sigma_b, x, n_\sigma) \cdot \dot{x} \tag{51}$$

$$\dot{M}_b = [A_4(\sigma_b, x, n_\sigma) + A_5(\sigma_b, x, n_\sigma) \cdot A_9(\sigma_b)] \cdot \dot{\sigma}_b + A_6(\sigma_b, x, n_\sigma) \cdot \dot{x} \tag{52}$$

Similarly, let us simplify the expressions for the axial force and bending moment resulting from stresses in the reinforcement bars.

$$\dot{N}_s = [A_s \cdot A_7(\varepsilon_b, x) + A_s' \cdot A_{12}(\varepsilon_b, x)] \cdot \dot{\varepsilon}_b + [A_s \cdot A_8(\varepsilon_b, x) + A_s' \cdot A_{13}(\varepsilon_b, x)] \cdot \dot{x} \tag{53}$$

$$\begin{aligned} \dot{M}_s = & \left[A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot A_{12}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot A_7(\varepsilon_b, x) \right] \cdot \dot{\varepsilon}_b + \\ & + \left[A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot A_{13}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot A_8(\varepsilon_b, x) \right] \cdot \dot{x} \end{aligned} \tag{54}$$

Taking into account relation (47) in the last two equations, we obtain:

$$\begin{aligned} \dot{N}_s = & [A_s \cdot A_7(\varepsilon_b, x) + A_s' \cdot A_{12}(\varepsilon_b, x)] \cdot A_{10}(\sigma_b) \cdot \dot{\sigma}_b + [A_s \cdot A_8(\varepsilon_b, x) + A_s' \cdot A_{13}(\varepsilon_b, x)] \cdot \dot{x} + \\ & + [A_s \cdot A_7(\varepsilon_b, x) + A_s' \cdot A_{12}(\varepsilon_b, x)] \cdot A_{11}(\sigma_b) \end{aligned} \tag{55}$$

$$\begin{aligned} \dot{M}_s = & \left[A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot A_{12}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot A_7(\varepsilon_b, x) \right] \cdot A_{10}(\sigma_b) \cdot \dot{\sigma}_b + \\ & + \left[A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot A_{13}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot A_8(\varepsilon_b, x) \right] \cdot \dot{x} + \\ & + \left[A_s' \cdot \left(\frac{h}{2} - a_s' \right) \cdot A_{12}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s \right) \cdot A_7(\varepsilon_b, x) \right] \cdot A_{11}(\sigma_b) \end{aligned} \tag{56}$$

Taking into account the obtained relations (51), (52), (55), and (56), let us rewrite the equilibrium equations (41) and (42) in terms of the rates $\dot{\sigma}_b$ and $\dot{x}$:

$$\begin{gathered}\left[A_1(\sigma_b, x, n_\sigma) + A_2(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b) + A_s \cdot A_7(\varepsilon_b, x)\cdot A_{10}(\sigma_b) + A_s' \cdot A_{12}(\varepsilon_b, x)\cdot A_{10}(\sigma_b)\right]\cdot \dot{\sigma}_b + \\ + \left[A_3(\sigma_b, x, n_\sigma) + A_s \cdot A_8(\varepsilon_b, x) + A_s' \cdot A_{13}(\varepsilon_b, x)\right]\cdot \dot{x} = \\ = \dot{P} - A_s \cdot A_7(\varepsilon_b, x)\cdot A_{11}(\sigma_b) + A_s' \cdot A_{12}(\varepsilon_b, x)\cdot A_{11}(\sigma_b)\end{gathered} \tag{57}$$

$$\begin{gathered}\left[A_4(\sigma_b, x, n_\sigma) + A_5(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b) + A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{12}(\varepsilon_b, x)\cdot A_{10}(\sigma_b) - \right. \\ \left. - A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_7(\varepsilon_b, x)\cdot A_{10}(\sigma_b) - P\cdot\frac{\rho_0}{x}\cdot A_{10}(\sigma_b)\right]\cdot\dot{\sigma}_b + \\ + \left[A_6(\sigma_b, x, n_\sigma) + A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{13}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_8(\varepsilon_b, x) + P\cdot\rho_0\cdot\frac{\varepsilon_b}{x^2}\right]\cdot\dot{x} = \\ = \dot{P}\cdot(e + f) + P\cdot\frac{\rho_0}{x}\cdot A_{11}(\sigma_b, \varepsilon_b) + A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_7(\varepsilon_b, x)\cdot A_{11}(\sigma_b) - \\ - A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{12}(\varepsilon_b, x)\cdot A_{11}(\sigma_b)\end{gathered} \tag{58}$$

For brevity, let us introduce the following notation:

$$\begin{gathered}B_1(\sigma_b, \varepsilon_b, x, n_\sigma) = A_1(\sigma_b, x, n_\sigma) + A_2(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b) + A_s \cdot A_7(\varepsilon_b, x)\cdot A_{10}(\sigma_b) + A_s' \cdot A_{12}(\varepsilon_b, x)\cdot A_{10}(\sigma_b), \\ B_2(\sigma_b, \varepsilon_b, x, n_\sigma) = A_3(\sigma_b, x, n_\sigma) + A_s \cdot A_8(\varepsilon_b, x) + A_s' \cdot A_{13}(\varepsilon_b, x), \\ C_1(\sigma_b, \varepsilon_b, x, n_\sigma) = \dot{P} - A_s \cdot A_7(\varepsilon_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b) + A_s' \cdot A_{12}(\varepsilon_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b), \\ B_3(\sigma_b, \varepsilon_b, x, n_\sigma) = A_4(\sigma_b, x, n_\sigma) + A_5(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b) + A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{12}(\varepsilon_b, x)\cdot A_{10}(\sigma_b) - \\ - A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_7(\varepsilon_b, x)\cdot A_{10}(\sigma_b) - P\cdot\frac{\rho_0}{x}\cdot A_{10}(\sigma_b), \\ B_4(\sigma_b, x, \varepsilon_b, n_\sigma) = A_6(\sigma_b, x, n_\sigma) + A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{13}(\varepsilon_b, x) - A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_8(\varepsilon_b, x) + P\cdot\rho_0\cdot\frac{\varepsilon_b}{x^2}, \\ C_2(\sigma_b, \varepsilon_b, x, f) = \dot{P}\cdot(e + f) + P\cdot\frac{\rho_0}{x}\cdot A_{11}(\sigma_b, \varepsilon_b) + A_s \cdot \left(\frac{h}{2} - a_s\right)\cdot A_7(\varepsilon_b, x)\cdot A_{11}(\sigma_b) - \\ - A_s' \cdot \left(\frac{h}{2} - a_s'\right)\cdot A_{12}(\varepsilon_b, x)\cdot A_{11}(\sigma_b).\end{gathered} \tag{59}$$

Then, relations (57) and (58) can be written in a more compact form as follows:

$$B_1(\sigma_b, x, n_\sigma)\cdot\dot{\sigma}_b + B_2(\sigma_b, x, n_\sigma)\cdot\dot{x} = C_1(\sigma_b, x, \varepsilon_b) \tag{60}$$

$$B_3(\sigma_b, x, n_\sigma)\cdot\dot{\sigma}_b + B_4(\sigma_b, x, n_\sigma)\cdot\dot{x} = C_2(\sigma_b, x, \varepsilon_b) \tag{61}$$

Solving this system with respect to the derivatives, we obtain:

$$\dot{\sigma}_b = F_1(\sigma_b, x, n_\sigma, \varepsilon_b) \tag{62}$$

$$\dot{x} = F_2(\sigma_b, x, n_\sigma, \varepsilon_b) \tag{63}$$

Here, the following notations are introduced:

$$F_1(\sigma_b, x, n_\sigma, \varepsilon_b) = \frac{C_1(\sigma_b, x, \varepsilon_b)\cdot B_4(\sigma_b, x, n_\sigma) - C_2(\sigma_b, x, \varepsilon_b)\cdot B_2(\sigma_b, x, n_\sigma)}{B_1(\sigma_b, x, n_\sigma)\cdot B_4(\sigma_b, x, n_\sigma) - B_2(\sigma_b, x, n_\sigma)\cdot B_3(\sigma_b, x, n_\sigma)} \tag{64}$$

$$F_2(\sigma_b, x, n_\sigma, \varepsilon_b) = \frac{C_2(\sigma_b, x, \varepsilon_b)\cdot B_1(\sigma_b, x, n_\sigma) - C_1(\sigma_b, x, \varepsilon_b)\cdot B_3(\sigma_b, x, n_\sigma)}{B_1(\sigma_b, x, n_\sigma)\cdot B_4(\sigma_b, x, n_\sigma) - B_2(\sigma_b, x, n_\sigma)\cdot B_3(\sigma_b, x, n_\sigma)} \tag{65}$$

Thus, the main system of differential equations for the problem under consideration is reduced to the normal form as follows:

$$\begin{gathered}\dot{\sigma}_b = F_1(\sigma_b, x, n_\sigma, \varepsilon_b) \\ \dot{x} = F_2(\sigma_b, x, n_\sigma, \varepsilon_b)\end{gathered}$$

$$
\begin{aligned}
\dot{\varepsilon}_b &= F_3(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{n}_\sigma &= F_4(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{f} &= F_5(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{\sigma}_s &= F_6(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{N}_b &= F_7(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{N}_s &= F_8(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{M}_b &= F_9(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{M}_s &= F_{10}(\sigma_b, x, n_\sigma, \varepsilon_b)\\
\dot{\sigma}'_s &= F_{11}(\sigma_b, x, n_\sigma, \varepsilon_b)
\end{aligned}
\tag{66}
$$

In the above expressions, the following notations are introduced for the right-hand sides:

$$
\begin{aligned}
&F_3(\sigma_b, x, n_\sigma, \varepsilon_b) = A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) + A_{11}(\sigma_b, \varepsilon_b)\\
&F_4(\sigma_b, x, n_\sigma, \varepsilon_b) = A_9(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b),\\
&F_5(\sigma_b, x, n_\sigma, \varepsilon_b) = A_{10}(\sigma_b)\cdot\frac{\rho_0}{x}\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) - \rho_0\cdot\frac{\varepsilon_b}{x^2}\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) + A_{11}(\sigma_b, \varepsilon_b)\cdot\frac{\rho_0}{x}\\
&F_6(\sigma_b, x, n_\sigma, \varepsilon_b) = A_7(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) + A_8(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) + A_7(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b)\\
&F_7(\sigma_b, x, n_\sigma, \varepsilon_b) = \{A_1(\sigma_b, x, n_\sigma) + A_2(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b)\}\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) + A_3(\sigma_b, x, n_\sigma)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b)\\
&F_8(\sigma_b, x, n_\sigma, \varepsilon_b) = A_s\cdot A_7(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) +\\
&\quad + A_s\cdot A_8(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) + A_s\cdot A_7(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b) +\\
&\quad + A'_s\cdot A_{12}(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) +\\
&\quad + A'_s\cdot A_{13}(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) + A'_s\cdot A_{12}(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b)\\
&F_9(\sigma_b, x, n_\sigma, \varepsilon_b) = \{A_4(\sigma_b, x, n_\sigma) + A_5(\sigma_b, x, n_\sigma)\cdot A_9(\sigma_b)\}\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) + A_6(\sigma_b, x, n_\sigma)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b)\\
&F_{10}(\sigma_b, x, n_\sigma, \varepsilon_b) = -A_s\cdot\left(\frac{h}{2} - a_s\right)\cdot A_7(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) -\\
&\quad - A_s\cdot\left(\frac{h}{2} - a_s\right)\cdot A_8(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) - A_s\cdot\left(\frac{h}{2} - a_s\right)\cdot A_7(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b) +\\
&\quad + A'_s\cdot\left(\frac{h}{2} - a'_s\right)\cdot A_{12}(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) +\\
&\quad + A'_s\cdot\left(\frac{h}{2} - a'_s\right)\cdot A_{13}(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) +\\
&\quad + A'_s\cdot\left(\frac{h}{2} - a'_s\right)\cdot A_{12}(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b)\\
&F_{11}(\sigma_b, x, n_\sigma, \varepsilon_b) = A_{12}(\sigma_b, x)\cdot A_{10}(\sigma_b)\cdot F_1(\sigma_b, x, n_\sigma, \varepsilon_b) +\\
&\quad + A_{13}(\sigma_b, x)\cdot F_2(\sigma_b, x, n_\sigma, \varepsilon_b) + A_{12}(\sigma_b, x)\cdot A_{11}(\sigma_b, \varepsilon_b)
\end{aligned}
\tag{57}
$$

Thus, the problem of determining the stress–strain state of compressed reinforced concrete elements with a rectangular cross-section is reduced to solving a Cauchy problem for a nonlinear system of differential equations with variable coefficients. The problem is then solved numerically using the fourth-order Runge–Kutta method, as in the case of stress relaxation problems. A corresponding software module implementing the Runge–Kutta method has been developed, and various numerical experiments have been carried out using this module. For an element with the following parameters: $b = 0{,}4\ м$; $h = 0{,}6\ м$; concrete $B\,30$, $\eta_1 = 1{,}3$; $m_1 = 4{,}3$; $\eta_2 = 1{,}6$; $m_2 = 4{,}0$; reinforced on one side only with reinforcement $A\,400 \quad 4\varnothing\,28$, subjected to nominally concentric compression with a random eccentricity $e = 1{,}0\ sm$, and having a length of $l = 6{,}0\ m$ the analysis was performed. Under short-term static loading, this element was analyzed using the method described in Section 2, and its load-bearing capacity was determined to be

$P_{ult} = 3650,375\ kN$. The same element was then investigated under the action of a sustained compressive force $P = 0,75796 \cdot P_{ult} = 2766,848\ kN$. The creep parameters had the following values: $\gamma = 0,025\ \ 1/day$, $C_0 = 0,1078 \cdot 10^{-7} \left(kN/m^2\right)^{-1}$, $B = 0,6$. As a result of the numerical analysis performed, it was found that the influence of creep on the deformation behavior of the compressed member is quite significant. For the given value of the external compressive force, the stress in the extreme compressed fiber of the concrete at the initial moment was equal to $\sigma_b = 0,8 \cdot R_b$. Due to creeping, this stress increased by only 0.07%. Despite this small change, the stress in the reinforcement increased by 17.67%. The maximum deflection of the element at the moment of loading was equal to
$f = 0,00242\ m$, while by the end of the creep process it reached $f = 0,00309\ m$, which corresponds to an increase of 27.69% in the maximum deflection due to creep. To study the influence of the reinforcement ratio, the same element was analyzed assuming symmetrical reinforcement. The initial value of the compressive force was selected such that, at the moment of loading, the stress in the extreme compressed concrete fiber, as in the first case, was equal to 80% of the compressive strength, i.e $P = 0,72850 \cdot P_{ult} = 3584,202\ kN$. The results of the analysis showed that creep practically did not affect the stress value in the most highly compressed reinforcement A_s'. Despite this, the maximum deflection of the considered element increased by 70.21% due to creep. All of the above once again emphasizes that the influence of concrete creep in compressed reinforced concrete elements must be properly considered.

Conclusions

1. The nonlinearity of concrete deformation, both under short-term static and long-term loading, has a significant effect on the stress–strain state of compressed reinforced concrete elements.
2. Due to concrete creep, the maximum deflection of a compressed element increases considerably, even under nominally concentric compression with accidental eccentricity, and this increase strongly depends on the reinforcement ratio.
3. Double reinforcement leads to a substantial increase in short-term load-bearing capacity. For the element considered, symmetric double reinforcement increased the short-term load-bearing capacity by 34.78% compared to single reinforcement.
4. For both single and double reinforcement, creep results in a significant increase in maximum deflection; for the example considered, this increase amounted to approximately 28% and 70%, respectively.
5. For mature concrete, the creep process stabilizes relatively quickly; in the examples considered, this period was approximately three months.
6. For young concrete, more advanced creep equations that account for the aging process of concrete should be used.

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