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ASYMPTOTIC ANALYSIS OF THE OSCILLATION FREQUENCY SUPPORTED BY INHOMOGENEOUS RIBS OF A CYLINDRICAL SHELL IN CONTACT WITH A LIQUID

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Abstract- Due to the demands of technology, the study of the dynamic behavior of structural elements, taking into account their adequate material properties, is of great importance. Modern mechanical engineering very often sets tasks for calculating thin-walled structures with mutually exclusive properties: on the one hand, the structures under study must combine high strength and reliability, and on the other, cost-effectiveness and lightness. For a successful combination of the above properties, the use of orthotropic materials and plastics in structures is quite justified.

Depending on the mechanical and thermal treatment, the type of technology, and the composition of the material, the property of uniformity and isotropy appears in the structural materials. On the other hand, such structures are exposed to the environment of various nature. In many cases, there is a need for reinforcement to improve the performance of such thin-walled structures.

Keywords: *Drainage installations, Porous material, The mixed energy method, Critical stress.*

1. INTRODUCTION

The [1] free vibrations of a cylindrical coating reinforced by inhomogeneous, isotropic, and inhomogeneous rings in contact with a liquid were studied. The Hamilton-Ostrogradsky variational principle was used to solve the problem. It is generally assumed that the heterogeneity of the rings used in fittings varies exponentially, and the heterogeneity of the cylindrical coating varies linearly in the direction of its thickness. The liquid was received perfectly. The rigid contact between the rods and the cylindrical cover is considered. With the help of contact conditions, a frequency equation is constructed, the roots are found by the numerical method, and characteristic curves are constructed. In [2], a similar task was performed for an inhomogeneous orthotropic cylindrical coating reinforced with inhomogeneous spindles. In [3], the issues of dance movements of cylindrical and conical coverings were investigated, taking into account the resistance of the external environment (Pasternak), and a comprehensive report was conducted, the results of which are presented in tables and curves of the relationship between the characteristic parameters. Moreover, in [4], solutions to issues of practical importance are presented, mainly taking into account the resistance of rectangular and round plates and cylindrical coatings of Fuss-Winkler and viscoelastic base. In their [5, 6] papers, a frequency equation was constructed to calculate the frequencies of free oscillation of cylindrical plates of a rectangular profile in contact with a viscoelastic medium, the roots were found, and the influence of physical, mechanical, geometric, and inhomogeneities, viscosity parameters, and geometric hole sizes characterizing the cylindrical plate and the external environment on these frequencies was studied. The analysis shows that the dancing of an inhomogeneous cylindrical shell reinforced with inhomogeneous rods in dynamic contact with a moving liquid has not been studied.

The presented article considered just such a question - free vibrations of a cylindrical coating reinforced with inhomogeneous, isotropic, and inhomogeneous rods in contact with a liquid. The Hamilton-Ostrogradsky variational principle was used to solve the problem. It is generally assumed that the heterogeneity of the rods used in fittings varies exponentially, and the heterogeneity of the cylindrical coating varies linearly in the direction of its thickness. The liquid was received perfectly.

2. FORMULATION OF THE PROBLEM

The rigid contact between the rods and the cylindrical shell is considered. With the help of contact conditions, a frequency equation is constructed, the roots are found by the numerical method, and characteristic curves are constructed.

Suppose that in contact with an ideal liquid, an inhomogeneous orthotropic cylindrical shell is reinforced along the axis with inhomogeneous rods (Fig.1). According to the Hamilton-Ostrogradsky variation principle [7]:

$$\delta\int_{t_0}^{t_1}\left(K - W - A_m\right)dt = 0\,. \qquad (1)$$

$$K = V_k + \sum_{i=1}^{k_1} K_i + \sum_{j=1}^{k_2} K_j\,. \qquad (2)$$

$$W = V_p + \sum_{i=1}^{k_1} \Pi_i + \sum_{j=1}^{k_2} \Pi_j . \qquad (3)$$

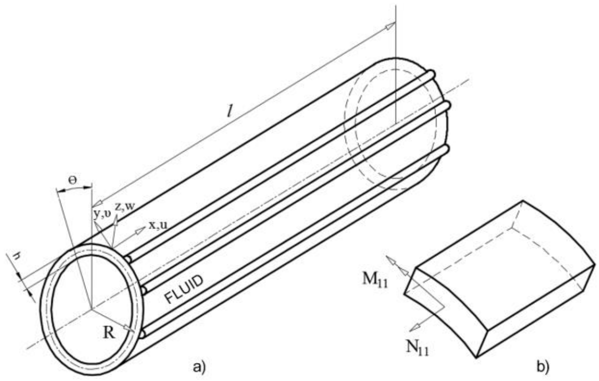

Figure 1. Inhomogeneous orthotropic cylindrical shell supported by inhomogeneous rods in contact with liquid.

To take into account the heterogeneity of the cylindrical coating of the spindles, we consider that the coating is a function of coordinates in the direction generating the elastic modulus and density of the spindles. Thus, the expressions of their energy will look like this:

$$V_p = \frac{Rh}{2} \iint \left(\sigma_{11}\varepsilon_{11} + \sigma_{12}\varepsilon_{12} + \sigma_{22}\varepsilon_{22} \right) ds. \qquad (4)$$

$$V_k = \int_0^{2\pi}\int_0^{l} \rho(x) \left(\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial \vartheta}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) dx d\varphi . \qquad (5)$$

$$\Pi_i = \frac{1}{2} \int_0^{l} \left[\tilde{E}_i(x) F_i \left(\frac{\partial u_i}{\partial x} \right)^2 + \tilde{E}_i(x) J_{xi} \left(\frac{\partial^2 \vartheta_i}{\partial x^2} \right)^2 + \tilde{E}_i(x) J_{zi} \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 + G_i(x) J_{kpi} \left(\frac{\partial \varphi_{kpi}}{\partial x} \right)^2 \right] dx. \qquad (6)$$

$$K_i = \int_0^{l} \tilde{\rho}_i(x) F_i \left[\left(\frac{\partial u_i}{\partial t} \right)^2 + \left(\frac{\partial \vartheta_i}{\partial t} \right)^2 + \left(\frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left(\frac{\partial \varphi_{kpi}}{\partial t} \right)^2 \right] dx. \qquad (7)$$

$$A_m = -\int_0^{l}\int_0^{2\pi} q_z w dx d\varphi. \qquad (8)$$

A cylindrical shell reinforced with spindles is a system consisting of a cylindrical shell and spindles rigidly attached to it along coordinate lines. It is assumed that the coordinate axes coincide with the lines of curvature of the cylindrical coating head, while the spindles are in rigid contact with the coating along these lines. Thus, the following conditions are met between the cylindrical coating and the spindles [7]:

$$u_i(x) = u(x, y_i),\ \vartheta_i(x) = \vartheta(x, y_i),\ w_i(x) = w(x, y_i) \tag{9}$$

$$\varphi_i(x) = \varphi_1(x, y_i); \varphi_1(x, y_i) = -\left.\frac{\partial w}{\partial x}\right|_{y=y_i}. \tag{10}$$

The pressure p created in the liquid is expressed in the following expression [8]:
Here

$$\Phi_{mk} = \begin{cases} I_k(\beta r) / I_k^{'}(\beta R),\ \ M_1 < 1 \\ J_k(\beta_1 r) / J_k^{'}(\beta_1 R),\ \ M_1 > 1 \\ \dfrac{r^k}{kR^{k-1}},\ M_1 = 1 \end{cases}$$

$$M_1 = \frac{U + R\omega / \text{m}}{a_0}, \beta^2 = R^{-2}\left(1 - M_1^2\right)m^2,\ \beta_1^2 = R^{-2}\left(M_1^2 - 1\right)m^2,\ \beta_1^2 = R^{-2}\left(M_1^2 - 1\right)m^2.$$

I_k - k – th compilation modified Bessel function of the first kind, J_k - k-th compilation Bessel function of the first kind, a_0 - is the speed of sound propagation in a liquid., U - the speed of fluid movement, $\xi = \frac{x}{l}$, m – this is the wave number in the x-axis direction.

The following contact conditions are satisfied between the liquid and the cylindrical coating [8]:

$$\vartheta_r\big|_{r=R} = -\left(\omega \frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x}\right). \tag{9}$$

$$q_z = -p_{|r=R} \tag{10}$$

We assume that the Novye conditions at the edges of the cylindrical shell satisfy [9]:

$$\vartheta = w = M_{11} = N_{11} = 0\ \ (by: x = 0, x = l\ ) \tag{11}$$

Thus, the vibrations of a cylindrical coating reinforced by inhomogeneous shafts dynamically in contact with a liquid are a solution to the problem of the total energy of a structure consisting of a cylindrical coating reinforced by discretely distributed inhomogeneous shafts, in the inner region of which liquid flows. (7), (9), (10) contact and (11) are brought to a joint integration under boundary conditions.

3. SOLUTION METHOD

Let us look at the coverage offsets as follows:

$$\begin{aligned} u &= A\cos n\theta \cos\frac{m\pi}{\xi_1}\xi \sin\omega t \\ \vartheta &= B\sin n\theta \sin\frac{m\pi}{\xi_1}\xi \sin\omega t \\ w &= C\cos n\theta \sin\frac{m\pi}{\xi_1}\sin\omega t \end{aligned} \qquad (12)$$

If we use solutions (12), Formulas (3) and (4), and contact conditions (7), we obtain a system of homogeneous algebraic linear equations capable of expressing complex constructions. Since the resulting system (14) is a system of linear homogeneous algebraic equations, the presence of its nontrivial solution is a necessary and sufficient condition for its main determinant to be zero. As a result, we obtain the following frequency equation:

$$det\left\|a_{pq}\right\| = 0, p, q = 1, 2, 3 \qquad (15)$$

where A, B, C are unknown constants, ω is a sought-for frequency, Using the solution of system (4), by displacing the points of the median surface of the shell (15) and contact conditions (13), (14), we can determine the contact pressure q_z. We represent this expression in the form of:

$$q_z = q_z^{(0)} C\cos n\varphi \sin kx \sin\omega t.$$

Where, in the case of small inertial actions of the medium on the oscillation process of the system, $q_z^{(0)}$ has the form:

$$q_z^{(0)} = -\mu_s \Delta^{-1}\Big\{\Big(2\left(1-2v_s\right) I_n\left(k^*\right) + 2k^* I_n^{'}\left(k^*\right)\Big) k^2 \times$$

$$\times\left[2k^2\left(k^{-2}-n^2\right)\frac{I_n^{'}\left(k^*\right)}{I_n\left(k^*\right)} + 2n^2 k^*\right] -$$

$$-2\left(k^* I_n^{'}\left(k^*\right) - \left(k^2+n^2\right) I_n\left(k^*\right)\right) k^3 +$$

$$+\left[2\left(3-2v_s\right)\cdot k'\frac{I_n^{'}\left(k^*\right)}{I_n\left(k^*\right)} - 2n^2\right] +$$

$$+2n\left(I_n^{'}\left(k^*\right) - k' I_n^{'}\left(k^*\right)\right) k^{-3} \times \left[2\left(3-2v_s\right) k'\frac{I_n^{'}\left(k^*\right)}{I_n\left(k^*\right)} - 2n^2\right]\Big\}.$$

When the inertial actions of the medium on the oscillation process of the system is significant, $q_z^{(0)}$ has the form of:

$$\left(-n^2 + n\dot{\gamma}_t \frac{I_n^{'}\left(\dot{y}_t\right)}{I_n\left(\dot{y}_i^{'}\right)} + \frac{v_s}{1-2v_s} n\gamma_t\left(\dot{y}_t - \dot{y}_t^{'}\cdot\frac{I_n^{'}\left(\dot{y}_t^*\right)}{I_n\left(\dot{y}_t^*\right)}\right)\right) +$$

$$+\left(\frac{\dot{k}'\ddot{\gamma}_t}{\dot{\mu}_t}\frac{r_n^{'}\left(\dot{\gamma}_t\right)}{I_n\left(\dot{\gamma}_t\right)} + \dot{\gamma}_t^2 + n^2 - \frac{v_s}{1-2v_s}\frac{2\dot{k}'\dot{\gamma}_t}{\mu_t^*}\right).$$

$$q_i^{(0)} = \frac{E_s}{1+v_s} I_n\left(\gamma_i^*\right)\left[\frac{I_n\left(\dot{\gamma}_t^*\right)}{I_n\left(\dot{\gamma}_1^*\right)}\left(-\dot{n}_1\frac{I_n^*\left(\dot{\gamma}_1^*\right)}{I_n\left(\dot{v}_1^*\right)} + \gamma_i^2 + n^2 - \frac{v}{1-2v_s}\mu^2\right)\times\right.$$

$$\times\frac{-n^2 k^{*2}\dot{\mu}_t^* + \dfrac{R^4 k^{*3}\gamma_t^*}{\mu_t^*}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_t^*\right)}}{-\dfrac{k^{*3}\dot{\gamma}_1^*\gamma_t^2}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_i^*\right)}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_t^*\right)}} + \frac{2nk^*\dot{\gamma}_l^*\mu_t^*\dfrac{I_n^{'}\left(\gamma_i^*\right)}{I_n\left(\gamma_i^*\right)} + \dfrac{2nk^{*3}\dot{\gamma}_t^*}{\mu_t}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\dot{\gamma}_t^*\right)}}{-\dfrac{k^3\dot{\gamma}_i^*\gamma_t^2}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_i^*\right) I_n^{'2}\left(\gamma_t^*\right)}{I_n\left(\gamma_i^*\right)}\dfrac{I_n^2\left(\dot{\gamma}_t\right)}{}}.$$

Where I_n is modified n th order Bessel function of first kind, k, n, γ_e, γ_t are wave numbers, $\gamma_e^2 = k^2 - \mu_e^2, k^* = kR,\ \gamma_t^2 = k^2 - \mu_t^2 \cdot \dot{\gamma}_t^* = \gamma_l R,\ \dot{\mu}_t = \mu_i R, \mu_i = \mu_l R.$

Completing the equations of motion of the shell (10), medium (11) by contact conditions (13), (14) we arrive at a contact problem on oscillations of the medrum-filled shell reinforced with longitudinal ribs. In other words, a problem of oscillations of an orthotropic shell with medium and reinforced with longitudinal ribs is reduced to joint integration of equations of theory of shells, medium subject to indicated conditions on their contact surface.

4. RESULTS AND CONCLUSIONS

For finding the approximate expressions of $q_z^{(0)}$, we will use asymptotic formulas for logarithmic derivative of the Bessel function $_n\left(x \le n; n \ge 1\right)$:

$$\frac{I_n^{'}\left(x\right)}{I_n\left(x\right)} \Rightarrow -\frac{n}{x}+\frac{x}{2n}.$$

Using formulas (16), (17) and (18) for $q_z^{(0)}$ we find: in the case of small inertial actions of the medium on oscillations process oscillation process of the system is significant, $q_z^{(0)}$ has the form of:

$$q_i^{(0)}=\frac{E_s}{1+v_s}I_n\left(\gamma_i^*\right)\left[\frac{I_n\left(\dot{\gamma}_t^*\right)}{I_n\left(\dot{\gamma}_1^*\right)}\left(-\dot{n}_1\frac{I_n^*\left(\dot{\gamma}_1^*\right)}{I_n\left(\dot{v}_1^*\right)}+\gamma_i^2+n^2-\frac{v}{1-2v_s}\mu^2\right)\times\right.$$

$$\times\frac{-n^2k^{*2}\dot{\mu}_t^*+\dfrac{R^4k^{*3}\gamma_t^*}{\mu_t^*}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_t^*\right)}}{-\dfrac{k^{*3}\dot{\gamma}_1^*\gamma_t^2}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\gamma_i^*\right)}\dfrac{I_n^{'2}\left(\gamma_t^*\right)}{I_n^2\left(\gamma_t^*\right)}}+\frac{2nk^*\dot{\gamma}_l^*\mu_t^*\dfrac{I_n^{'}\left(\gamma_i^*\right)}{I_n\left(\gamma_i^*\right)}+\dfrac{2nk^{*3}\dot{\gamma}_t^*}{\mu_t}\dfrac{I_n^{'}\left(\gamma_t^*\right)}{I_n\left(\dot{\gamma}_t^*\right)}}{-\dfrac{k^3\dot{\gamma}_i^*\gamma_t^2}{\mu_t^*}\dfrac{I_n^{'}\left(\gamma_i^*\right)I_n^{'2}\left(\gamma_t^*\right)}{I_n\left(\gamma_i^*\right)}I_n^2\left(\dot{\gamma}_t\right)}.$$

In the case of small inertial actions of the medium on oscillations process of the system.

$$q_z^{(0)} \approx \tilde{\chi}nE_s^*$$

and when the inertial actions of the medium on the oscillations process of the system are significant.

$$\tilde{q}_z^{(0)} \approx -\tilde{x}nE_s^*-2\frac{\rho_s^*}{n}\lambda.$$

Where in (19) and (20):

$$\lambda = \frac{\omega^2}{\omega_0^2}, \rho_s = \frac{\rho_s}{h_0\rho_0}, \tilde{\chi} = \frac{1 - v_{12}v_{21}}{2(1 + v_s)},$$
$$\dot{E}_s = \frac{E_s}{G_{12}h_s}, h\frac{h}{R}E_s/G_{12} \ll 1$$

After substitution of (15) and (19) in (10), we get a homogeneous system of linear algebraic equations that contains A, B, C, as unknowns, and whose nontrivial solution is possible only in the case when its determinant equals zero:

$$\det\|b_{iy}\| = 0 \ (i, j = 1,2,3).$$

Where,

$$b_{11} = -\left(a_1 + \gamma_c^{(1)}\right)k^2 - n^2 + \rho_1\lambda; b_{12} = (1 + a_{12})k^*n;$$
$$b_{13} = -a_{12}k^* + \delta_c^{(1)}k^{*3}; b_{21} = (1 + a_{12})k^*n, b_{22} = \lambda - a_2n^2;$$
$$b_{23} = na^2; b_{31} = a_{12}k^* + \delta_c^{(1)}k^{*3}; b_{32} = -a_2n;$$
$$b_{33} = a_2 + a^2\left(a_1k^{*4} + 2(a_{12} + 2)n^2k'^2 + a_{12}n^4 + \eta_c^{(1)}k^{\cdot 4}\right) -$$
$$-a_1\lambda + R^2\tilde{\chi}_n\dot{E}_1^*$$

In the open form, equation (21) has the form:

$$\lambda^3 + \alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3 = 0.$$

Where.

$$\alpha_1 = \rho_1^{-1}(\tilde{b}_{11} - \tilde{b}_{33} - a_2n^2);$$
$$\alpha_2 = \rho_1^{-2}\left[a_2n^2\tilde{b}_{33} + \left(\delta_c^{(1)2}k^{44} - a_{12}^2\right)k^2 - a_1n^2a_2^2 -\right.$$
$$\left.-\tilde{b}_{11}\tilde{b}_{33} - a_2n^2\tilde{b}_{11}\rho_1 - \rho_1(1 + a_{12})^2n^2k^{-2}\right];$$
$$\tilde{b}_{11} = -\left(a_1 + \gamma_c^{(1)}\right)k^2 - n^2;$$
$$\tilde{b}_{33} = a_2 + a^2\left(a_1k^4 + 2(a_{12} + 2)n^2k^2 + a_{12}n^4 + \eta_c^{(1)}k^{\cdot 4}\right) +$$
$$+R^2\tilde{x}nE_s^*;$$
$$\alpha_3 = \rho_1^{-2}\left[a_2n^2\tilde{b}_{11}\tilde{b}_{33} - b_{21}b_{32}b_{13} - b_{12}b_{23}b_{31} -\right.$$
$$\left.-a_2n^2k^2\left(\delta_c^{(1)2}k^4 - a_{12}^2\right) - n^2a_2^2\tilde{b}_{11} + \tilde{b}_{33}(1 + a_{12})^2n^2k^2\right]$$

When the inertial actions of the medium on the oscillations process of the system are significant, the equation with respect to λ takes the form:

$$\lambda^3 + \beta_1\lambda^2 + \beta_2\lambda + \beta_3 = 0.$$

Where,

$$\beta_1 = [\rho_1(\rho_1 + 2\rho_s^*/n)]^{-1}$$
$$[\tilde{b}_{11}\rho_1(\rho_1 + 2\rho_s^*/n) - \rho_1\tilde{b}_{33} - a_2n^2\rho_1(\rho_1 + 2\rho_s^*/n)];$$

$R = 0.16$ m; $h = 0.00045$ m; $v_2 = 0.19$; $v_1 = 0.11$;
$L_1 = 0.8$ m; $h_c = 0.1375 \times 10^{-1} R$; $\rho_0 = \rho_c = 7800 \text{kq/m}^3$;
$E_c = 6.67 \times 10^9 \Pi a$; $\rho_s = 2800 \text{kq/m}^3$; m = 8;
$\frac{J_y}{2\pi R^3 h} = 0.8289 \times 10^{-6}$; $\frac{J_{zi}}{2\pi R^3 h} = 0.13 \times 10^{-6}$;

$$\frac{J_{kpr}}{2\pi R^3 h} = 0.5305 \times 10^{-6}$$

The dependence of the frequency parameter on wave formation n in the circumferential direction found from equations (22) and (23) be means of (24) are depicted in Figure 1.

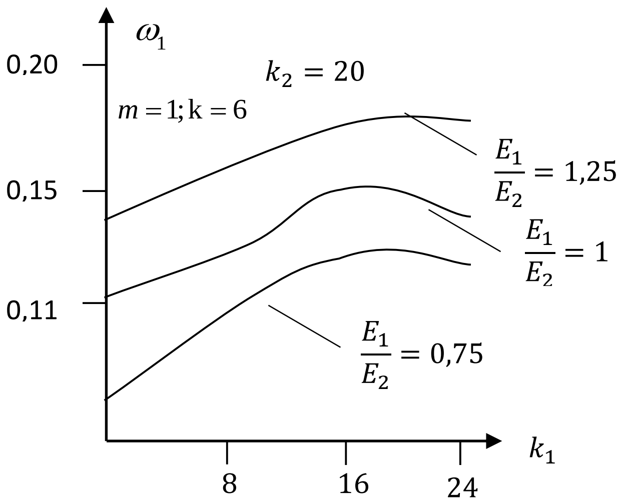

Figure 2. Dependence of the frequency parameter on the number of rods

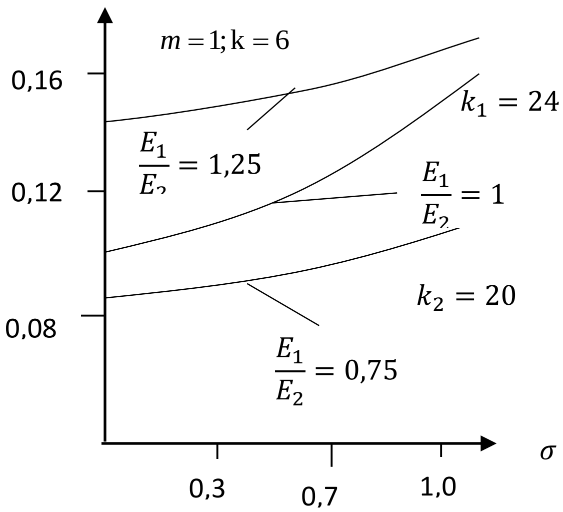

Figure 3. Dependence of the frequency parameter on the inhomogeneity parameter.

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