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THEORETICAL APPROACH TO THE AEROELASTIC STABILITY OF WINGS IN SUPERSONIC FLIGHT

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Abstract: *This study provides a theoretical investigation of how the wing–flow interaction at supersonic speeds (Mach > 1) affects aeroelastic stability. Variations in flow density and pressure, as well as the formation of shock waves in high-speed regimes, introduce additional aerodynamic loading on the wing, which may cause uncontrolled growth of structural vibrations. Among the aeroelastic instabilities that pose severe risks for high-speed flight, flutter is one of the most critical phenomena. In this study, the impact of airflow on the wing is evaluated using a simplified aerodynamic method. Meanwhile, the wing's structural response is analyzed through a classical beam model that captures its bending behavior. By coupling these two models, the governing equation of aeroelastic motion in a supersonic flow is derived, and the stability conditions characterizing the onset of flutter are obtained through harmonic analysis. The analytical results indicate that the flutter speed is highly sensitive to the wing's stiffness level, the distribution of mass per unit length, and the key flow parameters (density and Mach number). The findings provide a reliable theoretical basis for assessing aeroelastic safety in the design of wings operating in the supersonic flight regime.*

Keywords: *Supersonic flight, aeroelasticity, flutter, piston theory, critical stability condition, elastic deformation*

1. INTRODUCTION

The structural behavior of flight vehicles is determined by the interaction of aerodynamic, elastic, and thermal effects, and this interaction becomes particularly critical at high velocities, especially during supersonic flight. Throughout history, engineers and researchers have made significant efforts to develop high-performance and safe flight structures. In this context, understanding the aeroelastic response of structural components under supersonic and hypersonic conditions, as well as identifying the influence of thermal effects on this response, remains one of the essential research objectives. The interplay between airflow forces and the wing's structural response is a key factor in triggering aeroelastic instability.

Under supersonic flight conditions, structural components such as wings and fuselage elements experience substantial heating due to aerodynamic friction and high dynamic pressures. This thermal environment leads to the development of thermal stresses, and also alters the structural stiffness, natural vibration frequencies, and aerodynamic characteristics. Thermal stresses directly influence aeroelastic instabilities—particularly dynamic instabilities—by modifying the critical speeds and stability limits. Therefore, neglecting thermal effects in aeroelastic analyses can result in predictions that do not accurately represent real flight conditions.

The classification of different flight regimes is one of the essential steps in analyzing the structural and aerodynamic behavior of air vehicles. The Mach number, which measures how fast the aircraft moves compared to the surrounding sound speed, is an essential parameter, as its fluctuations directly influence the airflow behavior, pressure patterns, and the emergence of shock waves. Table 1 presents the Mach number ranges, regime names, and characteristic features associated with different flight regimes.

Table 1. Classification of flight regimes based on Mach number

Regime	**Mach number (M)**	**Description**	**Characteristics**
Subsonic	$M < 0.8$	Below the speed of sound	Smooth flow; compressibility effects are minimal
Transonic	$0.8 \leq M \leq 1.2$	Transition region	Portions of the flow become supersonic; shock waves begin to form
Supersonic	$1.2 \leq M \leq 5$	Above the speed of sound	Shock waves and compressibility effects dominate
Hypersonic	$M > 5$	Very high-speed regime	Intense thermal effects and gas ionization occur

Supersonic (Mach > 1) flight plays a significant role in modern aviation technology, where aerodynamic, structural, and thermal effects are strongly coupled within high-speed operating conditions. As the aircraft travels at supersonic speeds, shock waves develop in its surrounding airflow (Figure 1). These shock waves generate steep pressure gradients and notable density variations along the wing surface. Such changes influence the aerodynamic load distribution on the wing, thereby modifying the elastic response of the structure. In addition, the thermal distribution on the wing and fuselage may be non-uniform; the thermal gradients produced during flight can alter the local stiffness of the structure and introduce variations in its mechanical behavior. These combined effects are critically important, especially for air vehicles operating at high velocities.

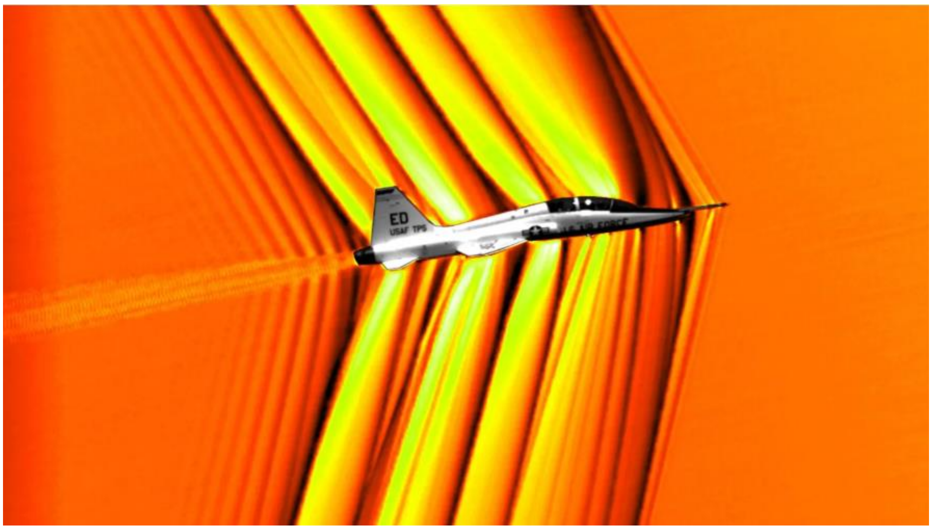

Figure 1. Shock waves generated around an aircraft during supersonic flight [5]

Aeroelastic instabilities are dynamic or static instability phenomena that arise from the interaction between the elastic properties of structural components and the aerodynamic loads acting on them. These phenomena become particularly critical at high velocities and under supersonic flight conditions. Aircraft wings, tail surfaces, and other structural elements undergo elastic deformation under aerodynamic forces. These deformations modify the aerodynamic load distribution, which in turn affects the structural response and overall flight performance. If this interaction is not properly controlled, severe instabilities and safety risks may occur within the structure.

Aeroelastic instabilities are generally categorized into two main groups: static instabilities and dynamic instabilities.

Static instabilities arise when the aerodynamic forces exceed the structural stiffness, a condition known as divergence. Divergence refers to the sudden and uncontrolled deformation of a structural element—such as a wing or control surface—beyond a certain load level. This type of instability poses a direct threat to the structural integrity and must be carefully considered during the design phase.

Dynamic instabilities, on the other hand, result from the interaction between the natural vibration frequencies of the structure and the aerodynamic forces, and this phenomenon is known as flutter. Flutter commonly occurs when the vibration modes of the wing couple with the aerodynamic feedback forces, leading to a resonant condition. This interaction causes excessive oscillations of the wing or structural component, eventually reaching a critically dangerous state for flight safety. The onset speed of flutter is directly influenced by structural stiffness, mode shapes, aerodynamic loading, and thermal effects.

It is important to note that equilibrium and stability are not the same concept: a system may be in force balance, yet still exhibit unstable behavior under small disturbances. Aeroelastic stability analysis of supersonic wings must take this distinction into account.

Previous research indicates that the aeroelastic behavior of supersonic wings depends on multiple parameters—such as flow velocity, density, wing elasticity, and mass distribution. Analytical approaches such as piston theory and the Euler–Bernoulli beam model are widely used to construct linear aeroelastic models. However, existing studies do not fully present an analytical relationship between flutter onset speed and the wing's structural and aerodynamic parameters.

The objective of this study is to analytically determine the critical stability condition and flutter onset speed for supersonic wings, and to establish a theoretical basis for ensuring aeroelastic stability. The study also provides design recommendations, such as increasing the effective stiffness of the wing and reducing flow density. This approach contributes to enhancing flight safety and optimizing wing design for supersonic aircraft.

Theoretical Model and Analytical Solution: Under supersonic flight conditions, the behavior of aircraft wings is governed by the interaction between aerodynamic forces and structural elastic forces. If the coupling between the wing's elastic deformation and the aerodynamic load is not properly managed, dynamic instabilities such as flutter may occur.

The aerodynamic force generated during wing vibration depends on the flow velocity and the deformation of the wing. In a supersonic flow, the linear aerodynamic model is expressed using piston theory [4]:

$$p = p_\infty + \frac{2p_\infty U_\infty}{\sqrt{M_\infty^2 - 1}}\left(\frac{\partial\omega}{\partial t} + U_\infty\frac{\partial\omega}{\partial x}\right) \tag{1}$$

p – the pressure exerted on the surface,
p_∞– the density of the undisturbed airflow,
U_∞ – the velocity of the incoming flow,
M_∞– the Mach number of the free-stream,
$\omega(x,t)$– transverse displacement of the wing surface.

Expression (1) indicates that, in a supersonic flow, the pressure variation depends both on time and on the deformation rate in the flow direction.

The elastic motion of the wing element is represented by the Euler–Bernoulli beam model:

$$EI\frac{\partial^4\omega}{\partial x^4} + m\frac{\partial^2\omega}{\partial t^2} = q(x,t) \tag{2}$$

E– elasticity modulus,
I – area moment of inertia,
m – mass per unit length,
$q(x,t)$ – aerodynamic load, defined as $q = p - p_\infty$.

Substituting piston theory into the structural equation yields:

$$EI\frac{\partial^4\omega}{\partial x^4} + m\frac{\partial^2\omega}{\partial t^2} = \frac{2p_\infty U_\infty}{\sqrt{M_\infty^2 - 1}}\left(\frac{\partial\omega}{\partial t} + U_\infty\frac{\partial\omega}{\partial x}\right) \tag{3}$$

Equation (3) describes the aeroelastic behavior of the wing in a supersonic flow.

Combining the elastic and aerodynamic equations—i.e., substituting (5) into (3)—the linear aeroelastic model is obtained:

$$EI\frac{d^4\omega}{dx^4} - m\omega^2 W = i\omega\frac{2p_\infty U_\infty}{\sqrt{M_\infty^2 - 1}}\left(W + \frac{U_\infty}{i\omega}\frac{dW}{dx}\right) \tag{4}$$

For analytical solution, the transverse displacement of the wing is assumed to be a harmonic vibration of sinusoidal form:

$$\omega(x,t) = W(x)e^{i\omega t} \tag{5}$$

With this assumption, the aeroelastic equation becomes a complex-valued expression, and a linear stability condition is derived.

For the system to remain stable, the real part of the general solution must not grow with time, which requires:

$$Im(\omega) \leq 0 \tag{6}$$

If $Im(\omega) > 0$, oscillations grow over time and flutter occurs.

At the stability boundary ($Im(\omega) = 0$), the approximate expression for the flutter onset velocity U_f is given as:

$$U_f = \sqrt{\frac{K_{eff}}{m} \cdot \frac{\sqrt{M_\infty^2 - 1}}{2p_\infty}} \tag{7}$$

K_{eff} – effective stiffness parameter of the wing.

The analytical model shows that the flutter onset speed is strongly dependent on wing structural stiffness, mass distribution, and flow parameters. Therefore, ensuring aeroelastic stability in supersonic wings requires careful optimization of material stiffness, mass distribution, and airfoil characteristics.

Discussion and Conclusion: The analytical model indicates that the flutter onset velocity is directly influenced by both structural and aerodynamic characteristics of the wing. Calculations show that increasing the effective stiffness of the wing raises the flutter onset speed, enhancing the wing's stability. Conversely, higher flow density and increased Mach number promote aeroelastic instability, thereby increasing the likelihood of flutter occurrence.

Solutions obtained from the analytical model and aeroelastic equations provide important insights into the behavior of supersonic wings. The results demonstrate that the onset of flutter is closely dependent on wing stiffness, mass distribution, and flow parameters. By increasing wing elasticity and optimizing mass distribution, the probability of flutter can be reduced, resulting in a more stable and reliable wing.

At the same time, elevated Mach numbers and increased flow density encourage aeroelastic instability, causing flutter to occur at lower velocities. These findings highlight the necessity of considering aerodynamic loading in conjunction with structural parameters for supersonic flight. The airfoil thickness and elastic properties are also critical; thicker and stiffer profiles are more resistant to aerodynamic loads.

The outcomes of the analytical model provide direct guidance for practical design decisions:

●Material selection should optimize both the modulus of elasticity and mass density.

●Mass distribution of the wing should be balanced, particularly based on vibration mode analysis.

●Airfoil thickness and structural stiffness must be chosen to ensure aeroelastic stability.

This study also demonstrates that a simple linear model based on piston theory serves as an effective tool for preliminary assessment of aeroelastic stability in supersonic wings. Analytical expressions allow for a rapid and accurate estimation of flutter onset speed and critical stability conditions.

Overall, the results establish a theoretical foundation for ensuring aeroelastic stability in supersonic wing design and contribute to enhancing flight safety. Future work should incorporate more complex aerodynamic effects, nonlinear material properties, and thermal influences to improve prediction accuracy under realistic flight conditions.

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