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STUDY ON THE FLEXURAL PERFORMANCE OF REINFORCED CONCRETE STRUCTURES STRENGTHENED WITH COMPOSITES

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Abstract: This study investigates the effect of reinforcing rectangular cross-sections and bending reinforced concrete elements with carbon fiber materials in the tensile zone. The main objective of this study was to determine the effect of this strengthening method on the stress-strain state and load-bearing capacity of the elements. For this purpose, an engineering calculation methodology was developed. The proposed methodology is based on the biaxial stress-strain relations of concrete, and unlike traditional approaches, it does not consider the tensile strength of the concrete. A special software module was developed to apply the calculation methodology and perform analysis. This module allows the study of the effects of various parameters, such as the thickness of the carbon fiber reinforcement layer, cross-sectional dimensions, and concrete class, on the mechanical behavior of the element. The numerical experiments clearly demonstrate the effectiveness of the strengthening technology. The results of this study showed that the load-bearing capacity of concrete elements reinforced with carbon fibers can be increased by up to 40%. This confirms that such a reinforcement method can be effectively applied in modern construction and repair work.

Keywords: reinforced concrete, stress-strain, load-bearing capacity, strengthening, carbon fiber, composite materials, flexure.

Introduction

Composite materials are widely used in the reinforcement and reconstruction of reinforced concrete structures. This is because of the high strength and technological advantages of these materials. Thus, composite materials allow for an increase in the load-bearing capacity of elements without increasing their cross-sectional dimensions. However, reinforcement using composite materials is not always possible. Owing to the high strength of composite materials, a very thin layer attached to the surface of the element can significantly increase the percentage of reinforcement, thereby changing the principle of operation of the element. Consequently, an ordinary reinforced element becomes an over-reinforced element with surface reinforcement. Therefore, construction standards do not consider reinforcing over-reinforced concrete elements with composite materials before reinforcement, and in these cases, recommend the use of other strengthening methods. In most cases, composite materials are manufactured with unidirectional working fibers. In some cases, materials with working fibers in two directions, that is, laminates, are produced. The tensile strength of carbon fiber laminates is higher than 1600 MPa and that of glass fiber-reinforced laminates is not lower than 520 MPa. Technology and engineering calculation methods for reinforcing reinforced concrete elements with composite materials are presented in detail in [1]. The use of unidirectional carbon tapes to reinforce RC elements has been described in [2]. The numerical values of the mechanical characteristics of the materials were also presented. Recently, in addition to normative documents on the strengthening of reinforced concrete structures, a sufficient number of scientific articles and books have been published [3]. In some cases, local standards are developed for certain

types of composite materials used in external reinforcements [4]. In general, calculation methods using nonlinear deformation models for both ordinary and reinforced concrete structures are being rapidly developing [5,6,7]. The Eurocode for reinforced concrete structures recommends the use of nonlinear models that accurately approximate real deformation diagrams of materials [8]. A comparative analysis of calculations performed using standards for reinforced concrete structures in different countries also shows that the application of a nonlinear deformation model is inevitable [9].

Mechanical Parameters of Composite Strengthening Materials

During the calculation of the ultimate limit states of reinforced concrete elements externally strengthened with composite materials, the design resistance of the laminate, considering the working condition factor, is determined as follows:

$$R_f = \gamma_{f1} \cdot \gamma_{f2} \cdot \gamma_{f3} \cdot R_{fn} \quad (1)$$

Here, γ_{f3} is the service condition factor of the laminate, which is taken as $\gamma_{f3} = 0{,}8$ for carbon fiber laminates under long-term loading, and $\gamma_{f3} = 0{,}3$ for glass fiber laminates. The factor γ_{f1}, representing the working condition, is determined based on the table provided in the following normative documents [1].

Table 1. Service condition factor of the laminate.

Service conditions of the structure	Types of composite	γ_{f3} is the service condition
Internal Locations	**Carbon fiber laminate**	0,95
	Glass fiber laminate	0,75
Outdoor Conditions	**Carbon fiber laminate**	0,85
	Glass fiber laminate	0,65
Aggressive Environment	**Carbon fiber laminate**	0,85
	Glass fiber laminate	0,5

According to normative documents, the modulus of elasticity for glass fiber laminates is $E_f \geq 15000\,MPa$, and for carbon fiber laminates, the modulus of elasticity is $E_f \geq 150000\,MPa$. The deformation limit of the laminate was determined as $\varepsilon_{f,ult} = \frac{R_f}{E_f}$ (2). According to normative documents, laminates are assumed to follow Hooke's law up to failure.

The main normative characteristics of composite materials are as follows [1]:

- Tensile Strength - R_{fn}
- Tensile Modulus of Elasticity- E_{fn}
- Ultimate Strain - $\varepsilon_{f,ult,n}$
- Coefficient of transverse expansion - μ_{fn}

The calculated values of the modulus of elasticity and coefficient of transverse expansion were taken to be equal to the normative values. The tensile strength of the laminate was calculated using the following formula [1]:

$$R_f = \frac{\gamma_{f1} \cdot \gamma_{f2} \cdot R_{fn}}{\gamma_f} \quad (3)$$

Here, γ_f is the reliability factor of the composite material. When calculating the second group of limit cases, $\gamma_f = 1$ is taken, when calculating the first group of limit cases, $\gamma_f = 1{,}2$ is taken for carbon fiber laminates, and for glass fiber laminates $\gamma_f = 1{,}8$, γ_{f1} is the operating condition factor of the composite material, γ_{f2} is the operating condition factor that takes into account the adhesion of the composite material to the concrete surface and is determined as follows [1]

$$\gamma_{f2} = \frac{1}{2{,}5 \cdot \varepsilon_{f,ult}} \cdot \sqrt{\frac{R_b}{n \cdot E_f \cdot t_f}} \le 0{,}9 \qquad (4)$$

In this equation, $\varepsilon_{f,ult}$ is the ultimate deformation of the composite material determined based $\gamma_{f2} = 1{,}0$ on Equation (2), n is the number of layers of the composite material, t_f is a dimensionless parameter and is taken as the numerical value of the thickness of one of the layers in millimeters, E_f is the value of the modulus of elasticity of the composite material in MPa, and R_b is the calculated compressive strength of concrete in MPa.

Calculation of Ultimate Forces for Rectangular Cross-Section Elements Strengthened with Composite Materials

Assume that, in general, a doubly reinforced rectangular flexural reinforced concrete element is strengthened in the tension zone with a laminate. In the ultimate state, as in the calculation of ordinary reinforced concrete elements, it is assumed that the element is in the failure stage. The tensile strength of the concrete was not considered. In the compression zone of the section, the distribution of compressive stresses is assumed to have a rectangular shape that is equal to the compressive strength of the concrete. It was assumed that the reinforcement bars reached their yield limits. Additionally, it was assumed that the laminate reached its failure limit. Therefore, the calculation scheme of the rectangular cross-sectional reinforced concrete element strengthened with laminate in the tension zone was adopted, as shown in Figure 1.

If we write the projection equation of all forces on the axis of the element, we obtain

$$R_b \cdot b \cdot x + R_{sc} \cdot A_s' - R_s \cdot A_s - R_f \cdot A_f = 0 \qquad (5)$$

Similarly, let us write the equation of motion of all forces relative to the center of gravity of the tensioned bar:

$$M = R_b \cdot b \cdot x \cdot \left(h_0 - \frac{x}{2}\right) + R_{sc} \cdot A_s' \cdot \left(h_0 - a_s'\right) + R_f \cdot A_f \cdot a_s \qquad (6)$$

By including the concept of the relative height of the compression zone of the cross-section, as in the calculation of conventional reinforced concrete elements, we can write from equation (5) that:

$$\xi = \frac{R_f \cdot A_f + R_s \cdot A_s - R_{sc} \cdot A_s'}{R_b \cdot b \cdot h_0} \qquad (7)$$

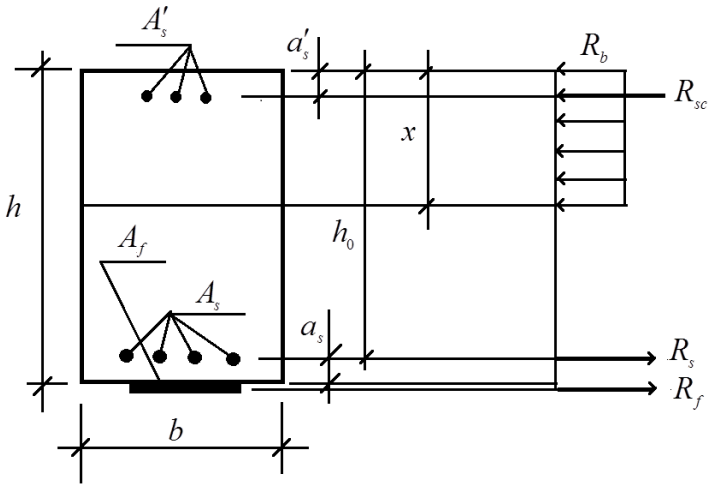

Figure 1. Calculation scheme of reinforced concrete element strengthened with laminate in the tension zone.

Then, from equation (6), we can write the formula for determining the load-bearing capacity of a section as follows:

$$M_{ult} = \alpha_m \cdot R_b \cdot b \cdot h_0^2 + R_{sc} \cdot A_s' \cdot \left(h_0 - a_s'\right) + R_f \cdot A_f \cdot a_s \qquad (8)$$

In this case, the moment coefficient is taken as $\alpha_m = \xi \cdot \left(1 - \frac{\xi}{2}\right)$, which is similar to the calculation of the ultimate limit states for ordinary reinforced concrete elements. Based on the obtained equations, we analyzed the calculation of the strengthened element using numerical examples.

Example. The cross-sectional dimensions of the reinforced concrete element were $b \times h = 40 \times 60\ sm$, concrete $B\,20$, and compressive strength $R_b = 11{,}5\ MPa$. The element was reinforced with $4 \varnothing 28$ reinforcement of class $A\,400$ with a total area $A_s = 24{,}63\ sm^2$ in the tension zone and $2 \varnothing 16$ reinforcement with a total area of $A'_s = 4{,}02\ sm^2$ in the compression zone, $R_s = R_{sc} = 350\ MPa$. We calculated the load-bearing capacity of this element. Let us assume $a_s = 4{,}4\ sm$ and $a'_s = 2{,}8\ sm$ during the calculations. The working height of section $h_0 = h - a_s = 60 - 4{,}4 = 55{,}6\ sm$. Then, from equation (7), we obtain for the relative height of the compression zone of the unreinforced element,

$$\xi = \frac{R_s \cdot A_s - R_{sc} \cdot A'}{R_b \cdot b \cdot h_0} = \frac{350 \cdot 24{,}63 - 350 \cdot 4{,}02}{11{,}5 \cdot 40 \cdot 55{,}6} = 0{,}282 < \xi_R = 0{,}533$$

Given that the moment coefficient $\alpha_m = \xi \cdot \left(1 - \frac{\xi}{2}\right) = 0{,}282 \cdot \left(1 - \frac{0{,}282}{2}\right) = 0{,}242$, the load-bearing capacity of the element can be determined using Equation (8).

$$M_{ult} = \alpha_m \cdot R_b \cdot b \cdot h_0^2 + R_{sc} \cdot A'_s \cdot \left(h_0 - a'_s\right) = 0{,}242 \cdot 11{,}5 \cdot 10^3 \cdot 0{,}4 \cdot 0{,}556^2 +$$
$$+ 350 \cdot 10^3 \cdot 4{,}02 \cdot 10^{-4} \cdot \left(0{,}556 - 0{,}028\right) = 475{,}30\ kN \cdot m$$

Now, let us assume that the load-bearing capacity of the element must be increased according to the reconstruction requirements. For this, we reinforced the element in the tension zone with a carbon fiber laminate of thickness $t_f = 1\ mm$, width $b_f = 20\ sm$, and tensile strength $R_f = 1600\ MPa$. Then, based on equation (7), we obtain for the relative height of the compression zone of the cross section:

$$\xi = \frac{R_f \cdot A_f + R_s \cdot A_s - R_{sc} \cdot A'_s}{R_b \cdot b \cdot h_0} = \frac{1600 \cdot 0{,}1 \cdot 20 + 350 \cdot 24{,}53 - 350 \cdot 4{,}02}{11{,}5 \cdot 40 \cdot 55{,}6} = 0{,}4072 < \xi_R = 0{,}5333$$

In this case, for the moment coefficient, we obtained

$\alpha_m = \xi \cdot \left(1 - \frac{\xi}{2}\right) = 0{,}4071 \cdot \left(1 - \frac{0{,}4071}{2}\right) = 0{,}3242$ Then, based on equation (8), for the load-bearing capacity of the section, we get

$$M_{ult} = \alpha_m \cdot R_b \cdot b \cdot h_0^2 + R_{sc} \cdot A'_s \cdot \left(h_0 - a'_s\right) + R_f \cdot A_f \cdot a_s =$$
$$= 0{,}3242 \cdot 11{,}5 \cdot 10^3 \cdot 0{,}4 \cdot 0{,}556^2 + 350 \cdot 10^3 \cdot 4{,}02 \cdot 10^{-4} \cdot \left(0{,}556 - 0{,}028\right) +$$
$$+ 1600 \cdot 10^3 \cdot 1 \cdot 10^{-3} \cdot 0{,}2 \cdot 0{,}044 = 549{,}39\ kN \cdot m$$

Thus, owing to laminate reinforcement, the load-bearing capacity of the section increased by $\frac{549{,}39 - 475{,}30}{475{,}30} \cdot 100\% = 15{,}59\ \%$. Note that if the laminate were glued across the entire width of the section, we would obtain from analogous calculations:

$$\xi = \frac{R_f \cdot A_f + R_s \cdot A_s - R_{sc} \cdot A'_s}{R_b \cdot b \cdot h_0} = \frac{1600 \cdot 0{,}1 \cdot 40 + 350 \cdot 24{,}53 - 350 \cdot 4{,}02}{11{,}5 \cdot 40 \cdot 55{,}6} = 0{,}5298 < \xi_R = 0{,}5333$$

$$\alpha_m = \xi \cdot \left(1 - \frac{\xi}{2}\right) = 0{,}5298 \cdot \left(1 - \frac{0{,}5298}{2}\right) = 0{,}3895$$

$$M_{ult} = \alpha_m \cdot R_b \cdot b \cdot h_0^2 + R_{sc} \cdot A'_s \cdot (h_0 - a'_s) + R_f \cdot A_f \cdot a_s =$$
$$= 0{,}3895 \cdot 11{,}5 \cdot 10^3 \cdot 0{,}4 \cdot 0{,}556^2 + 350 \cdot 10^3 \cdot 4{,}02 \cdot 10^{-4} \cdot (0{,}556 - 0{,}028) +$$
$$+ 1600 \cdot 10^3 \cdot 1 \cdot 10^{-3} \cdot 0{,}4 \cdot 0{,}044 = 656{,}33\ kN \cdot m$$

In this case, the load-bearing capacity increased by $\frac{656{,}33 - 475{,}30}{475{,}30} \cdot 100\% = 38{,}09\ \%$. This numerical example again shows that reinforcement with composite materials can significantly increase the load-bearing capacity of an element.

Calculation of rectangular cross-sectional reinforced concrete elements based on a nonlinear deformation model strengthened with composite materials in tension zone.

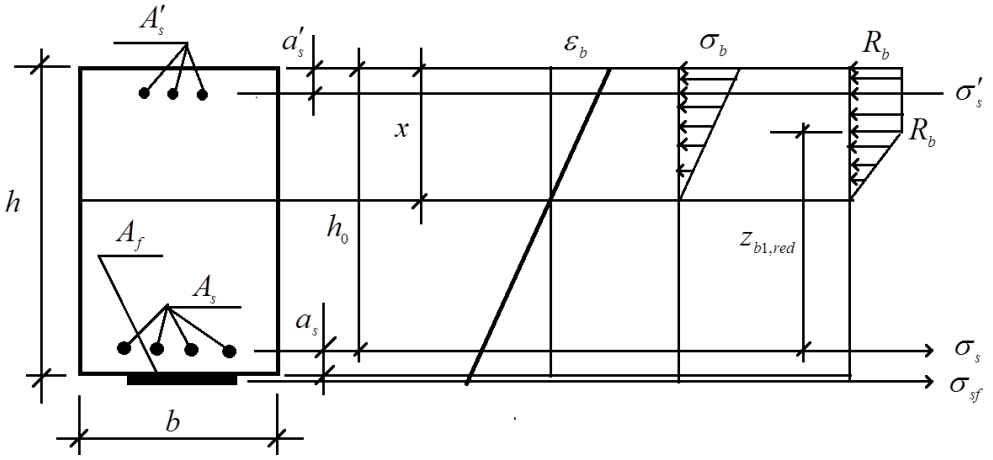

Figure 2. Reinforced Concrete Cross-Section Strengthened with Composite Material in the Tension Zone.

Assuming that the hypothesis of plane sections is true for a complex section, we can write the distribution of strain along the height of the section as

$$\varepsilon_z = \frac{\varepsilon_b}{x} \cdot (z + x - h_0) \qquad (9)$$

Then, for the distribution of compressive stress in concrete over the cross section, we can write:

$$\sigma_{bz}(\varepsilon_b, x) = \begin{cases} E_{b1,\,red} \cdot \frac{\varepsilon_b}{x} \cdot (z + x - h_0), & when\ \frac{\varepsilon_b}{x} \cdot (z + x - h_0) \le \varepsilon_{b1,red} \\ R_b, & when\quad \varepsilon_{b1,red} < \frac{\varepsilon_b}{x} \cdot (z + x - h_0) < \varepsilon_{b,\,ult} \end{cases} \qquad (10)$$

Based on this equation, for the internal normal force formed from the compressive stress in concrete, when the value of the deformation on the compressed surface of the cross-section satisfies the condition $\varepsilon_b \le \varepsilon_{b1,\,red}$, we obtain:

$$N_b(\varepsilon_b, x) = \frac{1}{2} \cdot b \cdot x \cdot \sigma_b(\varepsilon_b, x) \quad (11) \quad , \quad M_b(\varepsilon_b, x) = \frac{1}{2} \cdot b \cdot x \cdot \sigma_b(\varepsilon_b, x) \cdot \left(h_0 - \frac{x}{3}\right) \quad (12)$$

In the case where both axis of the biaxial diagram of concrete in compression are realized in the compression zone of the section, it is first necessary to determine the $z_{b1,\,red}$-coordinate. According to Figure 2

$$z_{b1,\,red} = x \cdot \left(\frac{\varepsilon_{b1,\,red}}{\varepsilon_b} - 1\right) + h_0 \qquad (13)$$

Then, for the internal normal force and bending moment resulting from the compressive stresses in the concrete, we obtain:

$$N_b = N_b(\varepsilon_b, x) = \frac{1}{2} \cdot R_b \cdot b \cdot (z_{b1,red} + x - h_0) + R_b \cdot b \cdot (h_0 - z_{b1,red}) = R_b \cdot b \cdot x \cdot \left(1 - \frac{\varepsilon_{b1,red}}{2\varepsilon_b}\right) \qquad (14)$$

$$M_b = M_b(\varepsilon_b, x) = \frac{1}{2}\cdot R_b \cdot b \cdot (z_{b1,red} + x - h_0)\cdot\left(h_0 - x + \frac{2}{3}\cdot(z_{b1,red} + x - h_0)\right) +$$
$$+ R_b \cdot b \cdot (h_0 - z_{b1,red})\cdot\left(h_0 - \frac{h_0 - z_{b1,red}}{2}\right) = \quad (15)$$
$$= \frac{1}{2}\cdot R_b \cdot b \cdot x \cdot \left(h_0 \cdot \left(2 - \frac{\varepsilon_{b1,red}}{\varepsilon_b}\right) - x \cdot \left(\frac{1}{3}\cdot\left(\frac{\varepsilon_{b1,red}}{\varepsilon_b}\right)^2 - \frac{\varepsilon_{b1,red}}{\varepsilon_b} + 1\right)\right)$$

From the similarity of triangles, we can write the deformation of the composite material, $\varepsilon_f = \frac{h - x}{x}\cdot\varepsilon_b$

We then obtain for the stress of the composite material, $\sigma_f = E_f \cdot \varepsilon_f = \frac{h - x}{x}\cdot E_f \cdot \varepsilon_b$

Therefore, for the internal normal force and bending moment generated by the composite material, we can write:

$$N_f = b_f \cdot t_f \cdot \sigma_f = E_f \cdot \varepsilon_f = b_f \cdot t_f \cdot E_f \cdot \frac{h - x}{x}\cdot\varepsilon_b \quad (16)$$

$$M_f = b_f \cdot t_f \cdot a_s \cdot \sigma_f = E_f \cdot \varepsilon_f = b_f \cdot t_f \cdot E_f \cdot a_s \cdot \frac{h - x}{x}\cdot\varepsilon_b \quad (17)$$

To calculate the internal normal force and bending moment formed by the stresses in the reinforcement bars, we first calculated their strains. According to the plane-section hypothesis, the strain of the compressed reinforcement is $\varepsilon'_s = \frac{x - a'_s}{x}\cdot\varepsilon_b$. Then, according to the deformation diagram of the reinforcement, we can write for the stress in this reinforcement,

$$\sigma'_s(\varepsilon_b, x) = \begin{cases} E'_s \cdot \varepsilon'_s = E'_s \cdot \frac{x - a'_s}{x}\cdot\varepsilon_b, & when \quad \frac{x - a'_s}{x}\cdot\varepsilon_b \le \varepsilon'_{s,ax} \\ R_{sc}, & when \quad \frac{x - a'_s}{x}\cdot\varepsilon_b > \varepsilon'_{s,ax} \end{cases} \quad (18)$$

Thus, the internal normal force and bending moment generated by this reinforcement are,

$N'_s(\varepsilon_b, x) = \sigma'_s(\varepsilon_b, x)\cdot A'_s$ (19) və $M'_s(\varepsilon_b, x) = \sigma'_s(\varepsilon_b, x)\cdot A'_s\cdot(h_0 - a'_s)$ (20)

Similarly, the strain of the tension bar is $\varepsilon_s = \frac{h_0 - x}{x}\cdot\varepsilon_b$ and the stress in this bar is:

$$\sigma_s(\varepsilon_b, x) = \begin{cases} E_s \cdot \varepsilon_s = E_s \cdot \frac{h_0 - x}{x}\cdot\varepsilon_b, & when \quad \frac{h_0 - x}{x}\cdot\varepsilon_b \le \varepsilon_{s,ax} \\ R_s, & when \quad \frac{h_0 - x}{x}\cdot\varepsilon_b > \varepsilon_{s,ax} \end{cases} \quad (21)$$

The internal normal force and bending moment resulting from this stress were,

$N_s(\varepsilon_b, x) = \sigma_s(\varepsilon_b, x)\cdot A_s$ və $M_s(\varepsilon_b, x) = 0$ (22)

Next, we express the equilibrium equations in the section under consideration:

$$N_b(\varepsilon_b, x) + N'_s(\varepsilon_b, x) - N_s(\varepsilon_b, x) - N_f(\varepsilon_b, x) = 0 \quad (23)$$

$$M = M_b(\varepsilon_b, x) + M'_s(\varepsilon_b, x) + M_f(\varepsilon_b, x) \quad (24)$$

The last two equations form the decisive system of equations for the problem under consideration and are solved based on the methodology proposed by Hajiyev [6,7]. This solution algorithm allows for the determination of the parameters characterizing the stress-strain state and load-bearing capacity with any desired level of accuracy.

Program module implementing the solution algorithm.

Below is the text of the program module in the Turbo Pascal ABC algorithmic language.

```
Program kompozit;
Label 11,22,33,44,55;
```

```
Var b,h,h0,x,epsb,sigmas1,sigmas,sigmab,rb,rs,epsb1red,epsax,
kappa,moment,nb,ns,mb,ms,xa,xb,xc,fa,fb,fc,ass1,ass,fs1,fs,ef,bf,tf,sigmaf,nf,mf:real;
var i,j,n:integer;
procedure beton(epsb,x,epsb1red,rb:real; var nb,mb:real);
label 555;  var z:real;
begin
if epsb<=epsb1red then begin nb:=b/2*rb/epsb1red*epsb*x;
mb:=b/2*rb/epsb1red*epsb*x*(h0-x/3); goto 555; end;
nb:=rb*b*x*(1-epsb1red/2/epsb);  z:=epsb1red/epsb;
mb:=rb/2*b*x*(h0*(2-z))-x*(z*z/2-z+1);  555:end;
Procedure armature(epsb,x,epsax,rs,ass,ass1,fs,fs1:real;
var sigmas1,sigmas,ns,ms:real);  var z:real;
begin
z:=epsb/x*(x-ass1); if z<=epsax then sigmas1:=rs/epsax*z else sigmas1:=rs;
z:=epsb/x*(x-h0); if abs(z)<=epsax then sigmas:=rs/epsax*z else sigmas:=-rs;
if fs1=0 then sigmas1:=0; ns:=sigmas1*fs1+sigmas*fs;
ms:=sigmas1*fs1*(h0-ass1);  end;
procedure laminat(epsb,x,ef,bf,tf:real; var sigmaf,nf,mf:real);
var z1:real;
begin
z1:=epsb/x*(ass-h0+x); sigmaf:=ef*z1; nf:=sigmaf*bf*tf;
mf:=-nf*ass;
end;
begin
b:=0.4; h:=0.6; ass1:=0.03; fs1:=0; ass:=0.044; fs:=0.002463;
rs:=350000; rb:=14500; epsb1red:=0.0015; epsax:=0.00175; h0:=h-ass;
ef:=150000000; bf:=0.2; tf:=0.001;
writeln('ILKIN VERILENLER');
writeln('b=',b:8:2,' m'); writeln('h=',h:8:2,' m');
writeln('as1=',ass1:8:2,' m'); writeln('as=',ass:8:2,' m');
writeln('As1=',fs1:12:8,' m2'); writeln('As=',fs:12:8,' m2');
writeln('Rs=',rs:8:2,' kN/m2'); writeln('Rb=',rb:8:2,' kN/m2');
writeln('epsb1red=',epsb1red:12:8); writeln('epsax=',epsax:12:8);
writeln('HESABLAMANIN NETICELERI');
writeln(' epsb x kappa moment sigmas');
j:=1; 44:epsb:=0.00035*j; xa:=0.0001*h0;
beton(epsb,xa,epsb1red,rb,nb,mb);
armature(epsb,xa,epsax,rs,ass,ass1,fs,fs1,sigmas1,sigmas,ns,ms);
laminat(epsb,xa,ef,bf,tf,sigmaf,nf,mf);
fa:=nb+ns+nf;  11:xb:=xa+0.001*h0;
beton(epsb,xb,epsb1red,rb,nb,mb);
armature(epsb,xb,epsax,rs,ass,ass1,fs,fs1,sigmas1,sigmas,ns,ms);
laminat(epsb,xb,ef,bf,tf,sigmaf,nf,mf);
fb:=nb+ns+nf;  if fa*fb>0 then begin xa:=xb; fa:=fb; goto 11; end;
33:if abs(xb-xa)<0.00001*h0 then goto 22;  xc:=(xa+xb)/2;
beton(epsb,xc,epsb1red,rb,nb,mb);
armature(epsb,xc,epsax,rs,ass,ass1,fs,fs1,sigmas1,sigmas,ns,ms);
laminat(epsb,xc,ef,bf,tf,sigmaf,nf,mf);
fc:=nb+ns+nf;
if fa*fc<0 then begin xb:=xc; fb:=fc; goto 33; end else
begin xa:=xc; fa:=fc; goto 33; end;
22:x:=(xa+xb)/2; beton(epsb,x,epsb1red,rb,nb,mb);
```

```
armature(epsb,x,epsax,rs,ass,ass1,fs,fs1,sigmas1,sigmas,ns,ms);
laminat(epsb,x,ef,bf,tf,sigmaf,nf,mf);
kappa:=epsb/x;  moment:=mb+ms+mf;
Writeln(epsb:12:5,'  ',x:12:4,'  ',kappa:12:5,'  ',moment:12:4,'  ',sigmas:12:1);
J:=j+1;  if  j<11  then  goto 44;  end.
```

Numerical Experiments. Let us consider the numerical examples using this program module. The cross-sectional dimensions of the reinforced concrete element were $b \times h = 40 \times 60\ sm$, concrete $B\,25$, and compressive strength $R_b = 14{,}5\ MPa$. The element was reinforced with $4 \varnothing 28$ reinforcement of class $A\,400$ with a total area $A_s = 24{,}63\ sm^2$ in the tension zone and $2 \varnothing 16$ reinforcement with a total area of $A'_s = 6{,}03\ sm^2$ in the compression zone, $R_s = R_{sc} = 350\ MPa$. We calculated the load-bearing capacity of this element. Let us assume $a'_s = 3\ sm$, $a_s = 4{,}4\ sm$, $\varepsilon_{b1,\,red} = 0{,}0015$ during the calculations.

The element was reinforced in the tension zone with a laminate with a modulus of elasticity $E_f = 150000000\ MPa$. The width of the laminate is $b_f = 20\ sm$, and the thickness is $t_f = 1\ mm$. The tensile strain state and load-bearing capacity of the element were determined. The calculations performed using the above software module provide the results shown in the tables below.

Table 1. Double-reinforced elements strengthened with carbon fibers.

$A'_s = 6{,}03\ sm^2$; $A_s = 24{,}63\ sm^2$; $b_f = 20\ sm$; $t_f = 1\ mm$

ε_b	$x,\ m$	$\chi,\ 1/m$	$M,\ kN \cdot m$	$\sigma_s,\ MPa$
0	0	0	0	0
0,00035	0,2569	0,00136	101,8395	-81,49
0,00070	0,2569	0,00272	203,6789	-162,98
0,00105	0,2569	0,00409	305,5184	-244,47
0,00140	0,2569	0,00545	407,3578	-325,96
0,00175	0,2251	0,00777	513,8457	-350
0,00210	0,2008	0,01046	531,4485	-350
0,00245	0,1922	0,01275	546,2652	-350
0,00280	0,1875	0,01493	560,0535	-350
0,00315	0,1851	0,01702	572,9991	-350
0,00350	0,1839	0,01903	585,2070	-350

Table 2. Double-Reinforced Non-Strengthened Element with Carbon Fiber

$A'_s = 6{,}03\ sm^2$; $A_s = 24{,}63\ sm^2$; $b_f = 0$; $t_f = 0$

ε_b	$x,\ m$	$\chi,\ 1/m$	$M,\ kN \cdot m$	$\sigma_s,\ MPa$
0	0	0	0	0
0,00035	0,2521	0,00139	100,0790	-84,37
0,00070	0,2521	0,00278	200,1581	-168,74
0,00105	0,2521	0,00416	300,2371	-253,10
0,00140	0,2521	0,00555	400,3161	-337,47
0,00175	0,2057	0,00851	473,7850	-350
0,00210	0,1749	0,01201	472,9061	-350
0,00245	0,1618	0,01515	472,8721	-350
0,00280	0,1533	0,01826	472,8751	-350
0,00315	0,1473	0,02138	472,8706	-350
0,00350	0,1429	0,02450	472,8775	-350

Table 3. Single-Reinforced Strengthened Element with Carbon Fiber

$A_s' = 0$; $A_s = 24{,}63\ sm^2$; $b_f = 20\ sm$; $t_f = 1\ mm$				
ε_b	$x,\ m$	$\chi,\ 1/m$	$M,\ kN \cdot m$	$\sigma_s,\ MPa$
0	0	0	0	0
0,00035	0,2746	0,00127	86,6967	-71,75
0,00070	0,2746	0,00255	173,3935	-143,49
0,00105	0,2746	0,00382	260,0902	-215,24
0,00140	0,2746	0,00510	346,7870	-286,99
0,00175	0,2739	0,00639	506,5495	-350
0,00210	0,2490	0,00843	519,0766	-350
0,00245	0,2356	0,01040	530,8932	-350
0,00280	0,2277	0,01230	542,0910	-350
0,00315	0,2228	0,01414	552,7378	-350
0,00350	0,2198	0,01592	562,9067	-350

Table 4. Single-Reinforced Non-Strengthened Element with Carbon Fiber

$A_s' = 0$; $A_s = 24{,}63\ sm^2$; $b_f = 0$; $t_f = 0$				
ε_b	$x,\ m$	$\chi,\ 1/m$	$M,\ kN \cdot m$	$\sigma_s,\ MPa$
0	0	0	0	0
0,00035	0,2700	0,00130	85,1294	-74,17
0,00070	0,2700	0,00259	170,2588	-148,33
0,00105	0,2700	0,00389	255,3882	-222,50
0,00140	0,2700	0,00519	340,5176	-296,67
0,00175	0,2601	0,00673	479,1660	-350
0,00210	0,2312	0,00908	479,1760	-350
0,00245	0,2142	0,01144	479,1749	-350
0,00280	0,2030	0,01379	479,1778	-350
0,00315	0,1951	0,01615	479,1726	-350
0,00350	0,1892	0,01850	479,1702	-350

The moment-curvature diagram (Figure 3) of the double-reinforced elements demonstrates a remarkable difference in the performance of the strengthened element against non-strengthened elements 1 and 2. Both diagrams follow a similar pattern up to the initial elastic phase of stiffness development of the element. However, the strengthened element can show an ultimate moment of approximately 585.21 kNm as compared to that for the nonstrengthened of 472.91 kNm, an increased capacity of approximately $\frac{585{,}21 - 472{,}91}{472{,}91} \cdot 100\% = 23{,}75\ \%$gain in the ultimate load-carrying capacity with increased curvature. This increased curvature can be considered an increase in ductility without a significant drop in performance. This increased performance can be related to the effectiveness of the carbon fiber laminate reinforcement in delaying the onset of cracking and subsequently redistributing the tensile stresses in the cross section. Thus, the strengthened element retains integrity and load-carrying capacity for a larger range of curvatures than its unstrengthened counterpart, thus showing the benefits of composite reinforcement with respect to the structural behavior of double-reinforced elements.

In the moment-curvature relationship for single-reinforced elements, as shown in Figure 4, it is clear that strengthened element (1) outperforms the non-strengthened element (2). The strengthened element reaches the ultimate bending moment of 562.91 kNm, whereas the plateaus of the non-strengthened element reach 479.18 kNm, corresponding to a $\frac{562{,}91 - 479{,}18}{479{,}18} \cdot 100\% = 17{,}47\ \%$ increase in capacity. In the elastic phase, a similar trajectory is observed for the two curves. However, owing to the high curvature, the non-strengthened element started to show early stiffness degradation compared to the strengthened element. These strengthened elements responded smoothly, and their ductility was characterized by their ability to resist larger deformations until

failure. This indicates that carbon fiber laminates are effective in compensating for the deficiency in the tensile strength of concrete in single-reinforced configurations. An increased curvature capacity also implies increased resistance against seismic or dynamic loading conditions, which makes carbon fiber reinforcement effective, reliable, and suitable for retrofitting and strengthening purposes.

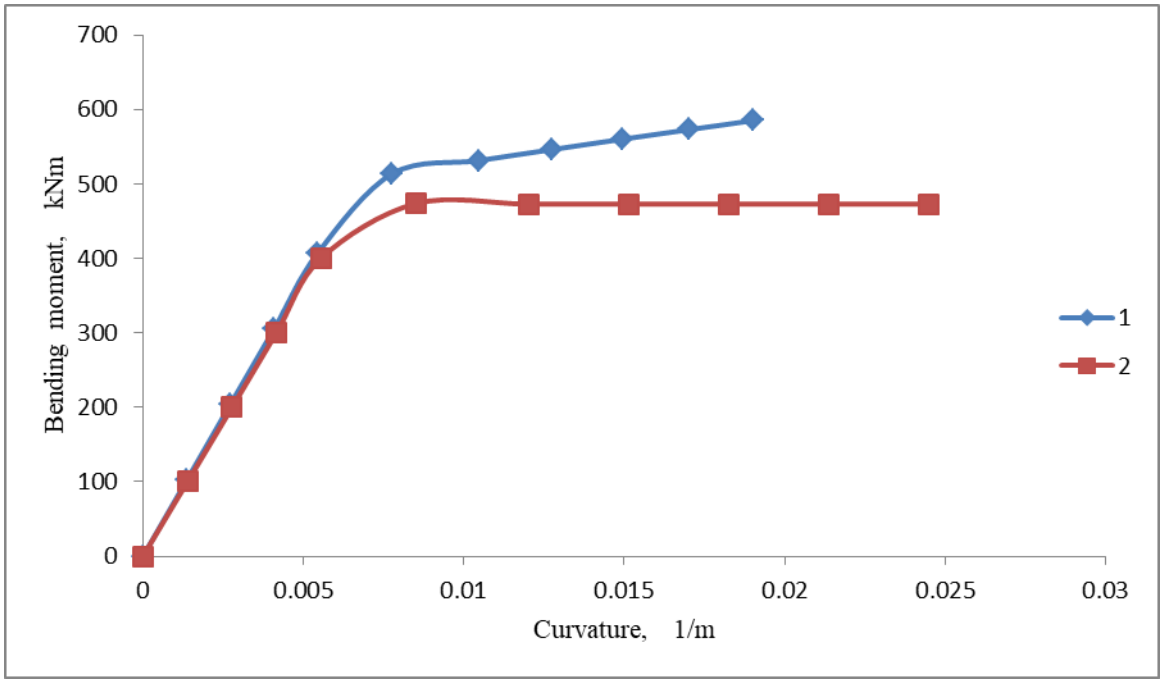

Figure 3. Moment-Curvature relationship for double-reinforced element. 1 – Strengthened Element 2 – Non-Strengthened Element

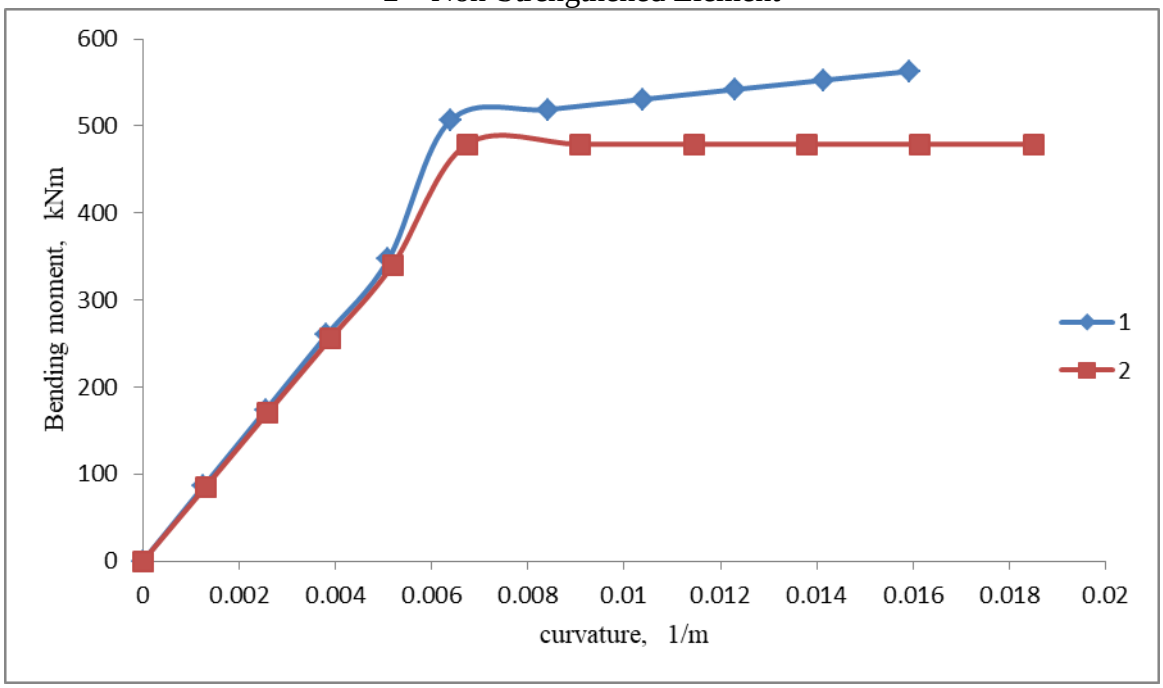

Figure 4. Moment-Curvature relationship for single-reinforced element. 1 – Strengthened Element, 2 – Non-Strengthened Element

The diagram of the strain variation in Figure 5 illustrates the behavior of the height of the compressive zone in the reinforced concrete elements with increased strain. Four cases were analyzed in this study. It can be observed from the test results that with an increase in the compressive face strain of the section, the height of the compressive zone decreased for all cases. However, the compressive zone height decreases at a slower rate in the case of strengthened elements reinforced with carbon fiber laminates than in the case of non-strengthened elements. This indicates an improved stress redistribution and efficient load-carrying capacity owing to the carbon

fiber reinforcement. In the case of double-reinforced elements, the strengthened configuration maintains a higher compressive zone height throughout the strain range than the non-strengthened configuration. At higher strain levels, the compressive zone height in the non-strengthened element decreases significantly, signaling an earlier failure, whereas the strengthened element continues to utilize the capacity of the concrete effectively. A similar trend was observed for the single-reinforced elements, although the strengthening effect of the carbon fiber reinforcement was much stronger in that case. The compressive zone height for the strengthened single-reinforced element remains considerably higher at all levels of strain, with differences exceeding 20% at the maximum strain values compared to the nonstrengthened counterpart. Carbon fiber laminates enhance structural performance by improving stress redistribution and ductility extension, thereby increasing the load-carrying capacity of the section. Strengthened elements exhibit improved resistance to strain-induced degradation of the compressive zone height, which is critical for maintaining the structural integrity under extreme loading conditions.

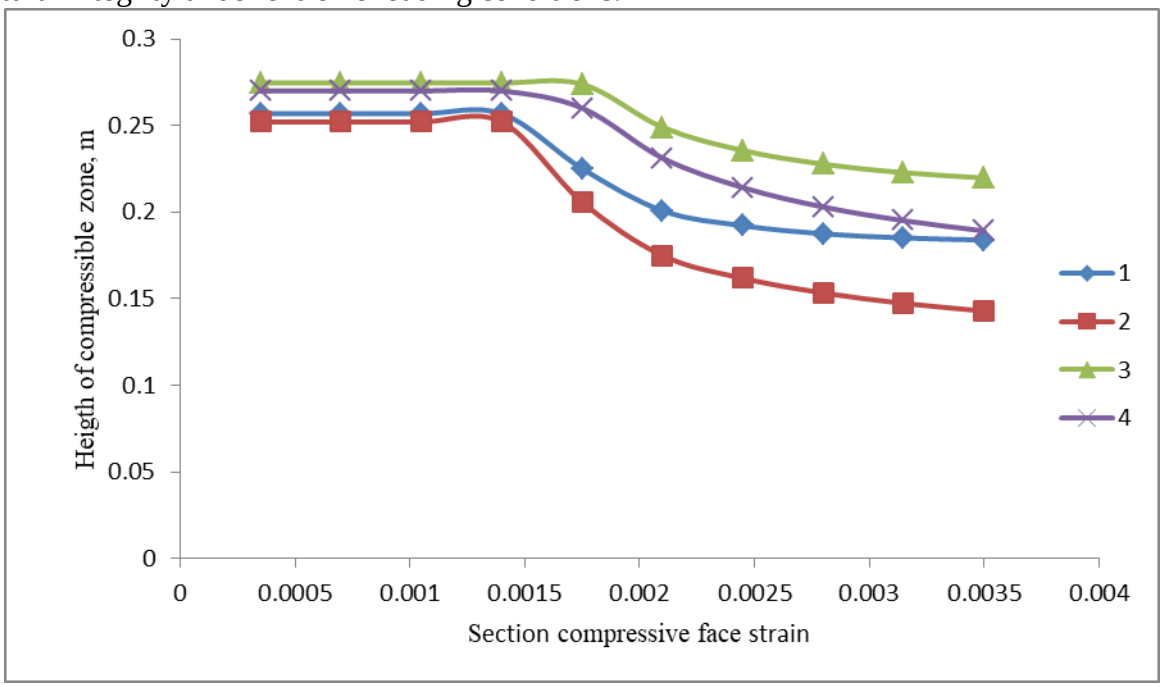

Figure 5. Strain variation diagram of compression zone depending on height compression face of section. 1 – double-reinforced strengthened element with carbon fiber, 2 – double-reinforced non-strengthened element with carbon fiber, 3 – single-reinforced strengthened element with carbon fiber, 4 – single-reinforced non-strengthened element with carbon fiber

Conclusion

This study presents a reliable methodology to assess the strucutral performance of reinforced concrete members whose tensile zone is strengthened with carbon fiber laminates. The most important results and their practical consequences are as follows.

- A methodology was developed to calculate the strength of normal sections in plain reinforced concrete elements using a bilinear stress-strain diagram of concrete under compression. This will improve the accuracy of the structural performance predictions.
- An effective method was developed for determining the stress-strain state and load-carrying capacity of reinforced concrete elements with rectangular cross sections strengthened in the tensile zone with high-strength carbon fibers. A module of the program was developed, which allowed the performance of this methodology in practical numerical analyses.
- The test results confirm that external strengthening with carbon fibers in the tension zone increases the load-carrying capacity of the reinforced concrete elements by more than 20%. This significant enhancement definitely proves the efficiency of strengthening with carbon fibers and completely justifies its recommendation as a reliable method for retrofitting structural elements to enhance their performance.

- The suggested calculation methodology opens perspectives for further development related to approximate engineering methods for the structural reconstruction of buildings and infrastructure. This code can be applied to a wide range of engineering tasks, owing to its flexibility and precision.
- The program modules developed will ensure that the proposed methodology has great potential for effective application to real-world problems, and it will allow engineers to confidently and efficiently solve practical design and retrofitting tasks.

As this research has shown, strengthening with carbon fiber is critical for making substantial improvements in structural performance by enhancing the level of reinforced concrete members. All the developed methodologies and tools will become fundamental knowledge for their eventual realization in engineering design and practice.

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