Azerbaijan University of Architecture and Construction
ISSN 2706-7726
Engineering Mechanics
Scientific and Technical Journal
E-mail: engineeringmechanics@azmiu.edu.az

September 2024	Issue 16	Volume 8	Number 2	Pages 36-42

STRESS-DEFORMATION STATE OF SHORT REINFORCED CONCRETE SUPPORTS WITH RING-SECTION IN COMPRESSION.

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Abstract: The article presents an effective methodology for determining the stress-strain state and load-bearing capacity of short reinforced concrete suppots with annular cross-sections using a nonlinear deformation model. When developing the solution methodology, the work of concrete in compression was described by the fractional-rational dependence proposed by the Eurocode. It was assumed that the reinforcing bars have the same resistance in tension and compression, and the work of the reinforcing bars was described by a symmetrical two-line diagram. By approximating the bent axis of the compressed bar with a half-wave of the sine, the solution of the problem was reduced to the solution of a system of nonlinear equations that determine the position of the neutral axis in the most stressed section and the level of deformation on the outer compressive surface of this section. After the level of deformation is assumed and the parameter that determines the position of the neutral axis is determined as the root of the equation with one unknown, the parameters characterizing the stress-strain state are easily determined. As a result of the calculations, the compressive force of the element and the corresponding maximum deflection are also determined, so the "load-deflection" graph, which is important in determining the load-bearing capacity, can be easily constructed. The proposed calculation method allows you to monitor the stress-strain state until the moment of complete collapse. The effectiveness of the proposed calculation method has been proven in numerical examples.

Keywords: *concrete, reinforcement, stress, deformation, load-bearing capacity.*

Introduction. Short reinforced concrete supports with an annular cross-section are widely used as the main load-bearing elements in various fields of construction [3]. One of the main reasons for the application of this type of elements as supports is their operation under conditional central compression rules, which allows using the maximum strength reserve of materials. The results of theoretical and experimental studies show that the load-bearing capacity of this type of reinforced concrete elements is mainly determined by their strength condition. Sometimes, depending on the value of flexibility, the load-bearing capacity can also be determined by the stability condition. This happens when the reduction arms of the "load-deflection" graphs of the compressed element are also realized. In [1], the authors analyzed the theory of calculation of reinforced concrete elements based on the norms of the Russian Federation and showed that the application of nonlinear models when establishing calculation methods, as proposed in the Eurocode, is inevitable and the development of the theory should be carried out in this direction. In [2], an effective methodology was developed for determining the stress-strain state and load-bearing capacity of the eccentrically compressed reinforced concrete elements with circular and annular cross-section. The developed methodology is general and allows determining the parameters characterizing the stress-strain state with any accuracy. In [5], the stress-strain states of bridges designed as cicular reinforced concrete elements during their collision with ships and barges were studied. The studies were conducted using laboratory tests and numerical modeling using the finite element method. It was shown that the proposed numerical calculation model correctly reflects the results of the tests, since it also takes into account the nonlinear behavior of materials. In [6], the stress-strain state of tubular concrete, that is, metal columns filled with concrete, depending on the class of concrete and metal, was studied. During experimental studies, the effect of the length of the element, the diameter of the metal pipe, the thickness of its wall, the compressive strength of concrete, etc. parameters on the load-bearing capacity was studied. As a result of the study, an empirical formula was proposed for determining the load-bearing capacity. In [7], the behavior of tubular concrete columns with circular cross-sections with different slots in central compression was studied. In addition to experimental studies, a nonlinear analysis was also performed numerically using the finite element method and it was shown that the results of the nonlinear analysis agree with the experimental results with sufficient accuracy. It has been shown that the multi-slotted cross-section can provide a 30% increase in load-bearing capacity compared to simle elements. In [8], tubular concrete elements with a circular cross-section were studied. The results of the experiments showed that since the wall thickness of the metal pipes is sufficiently small, local swelling of the pipes occurs during central compression. In the experimental "load-deflection" graphs, descending arms are also realized. As a result of the study, appropriate empirical formulas are proposed for determining the load-bearing capacity. It was noted that the load-bearing capacity can also be adjusted by adjusting the diameters of the inner and outer pipes and their thicknesses. In [9], centrally compressed columns with a circular cross-section were experimentally studied in various reinforcements. Reinforcement with external carbon fibers and reinforcement with indirect reinforcement were studied. It was shown that external reinforcement allows increasing the plasticity of the compressed element and increasing the equivalent load-bearing capacity by 30%. The mentioned brief analysis once again proves the importance of developing the calculation methodology for circular and annular support structures. The article mainly studies short elements, that is, elements whose overall load-bearing capacity is determined by the strength condition.

Problem statement. In the authors' previous studies, the deformation diagram of concrete in compression was approximated by the fractional-rational function proposed by the Eurocode, and the deformation diagram of reinforcement was approximated by a two-line diagram with a limited yield area, and the solution of the problem was reduced to the solution of a system of nonlinear algebraic equations. This system of equations is a nonlinear system of equations depending on the level of deformation on the compressive surface of the cross section and the parameter that determines the position of the neutral axis. In the general case, this system of equations is written as follows:

$$2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta,\zeta) + N_s^*(\beta,\zeta) = P \quad \text{və} \quad 2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta,\zeta) + M_s^*(\beta,\zeta) = P \cdot (e + f) \quad (1)$$

When approximating the equation of the axis of the support in longitudinal bending as $y(x) = f \cdot \sin \frac{\pi \cdot x}{l_0}$, the maximum deflection parameter arising during longitudinal bending can be expressed as $f = \rho_* \cdot \frac{\beta}{\xi}$; $\rho_* = \frac{\varepsilon_R \cdot l_0^2}{\pi \cdot R^2}$ (2) with the main parameters β and ξ included in the calculations, so these parameters can be determined as single-valued from the system (1), which is the main system of equations. For this, the force parameter can be eliminated from this system and the following equation can be written that connects these two parameters:

$$\Phi(\beta,\zeta) = 2 \cdot R^3 \cdot R_b \cdot M_b^*(\beta,\zeta) + M_s^*(\beta,\zeta) - \left(e + \rho_* \cdot \frac{\beta}{\xi}\right) \cdot \left[2 \cdot R^2 \cdot R_b \cdot N_b^*(\beta,\zeta) + N_s^*(\beta,\zeta)\right] = 0 \quad (3)$$

Solution to the problem. The equation (3) obtained above is the main solving equation of the problem under consideration. Since the interval of variation of the parameter β for all classes of concrete is known in this equation, $\beta \in [0; 1{,}75]$ the ξ parameter corresponding to the value adopted from (3) is determined as the positive root of the one-unknown transcendental equation. The values found allow us to easily calculate other parameters characterizing the stress-strain deformation state of the compressed element. Also, at the adopted value of deformation, the stresses in the reinforcement rods, the maximum deflection of the support, the value of the force compressing the support, which part of the external compressive force is received by the concrete and which part by the reinforcement rods, etc. allow us to find other parameters characterizing the stress-strain deformation state. For example, after the, β and ξ parameters are determined, the value of the compressive force can be determined from the first equation of dependence (1) and the maximum deflection corresponding to this force can be determined based on equation (2). Then, based on the relevant formulas given in the authors' work [4], the stresses in the reinforcement rods and other parameters can be calculated. Based on the obtained results, the ordinates of the graph expressing the "load-strain" dependence are determined. This dependence plays an important role in determining the load-bearing capacity of the element.

Block diagram of the solution algorithm. The block diagram of the described solution algorithm is given in Figure 1 below.

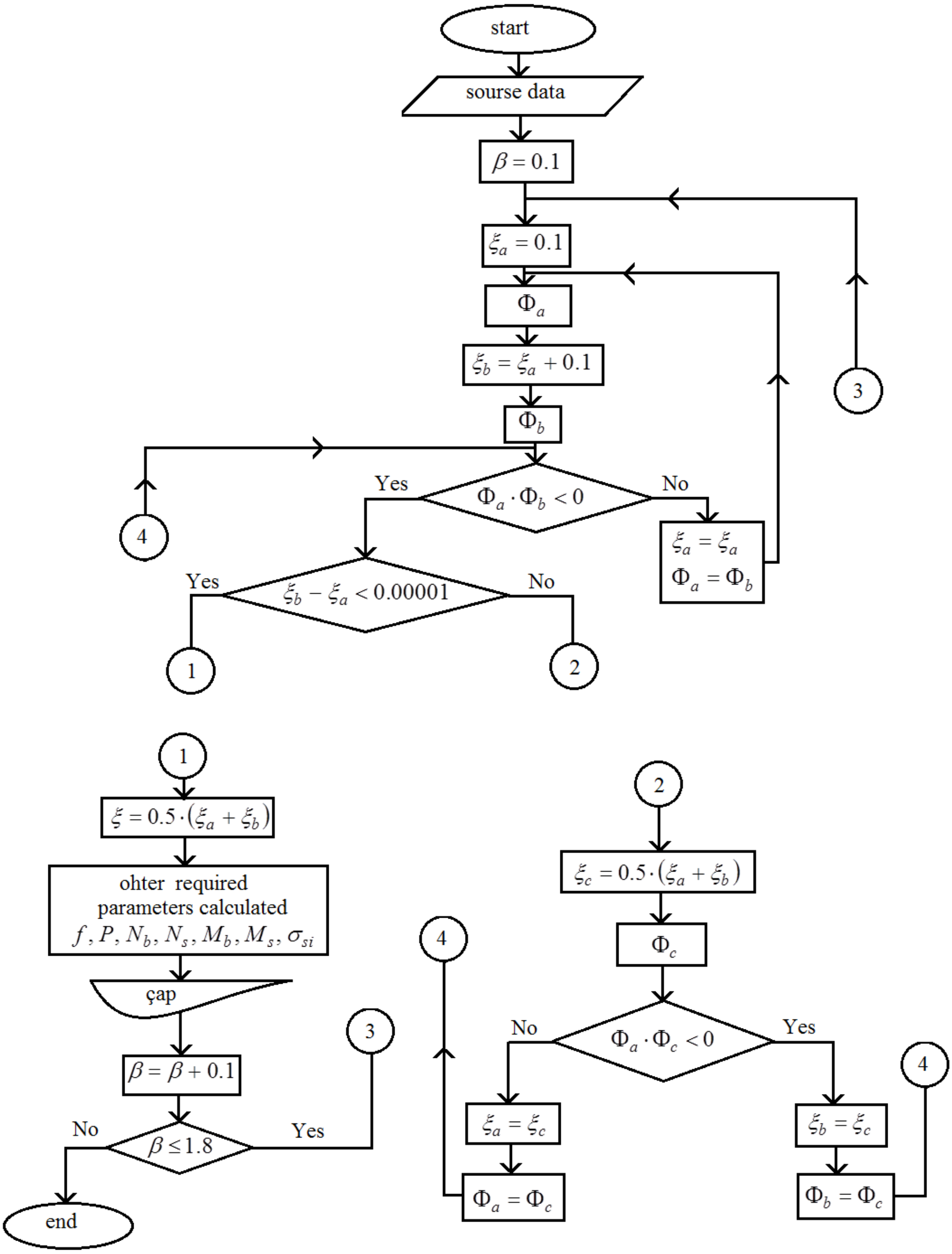


Fig. 1. Block diagram of the solution algorithm.

A software module that implements the solution. The software module that implements the solution. The text of the software module, written in the Turbo Pascal ABC algorithmic language based on this block-sem, is as follows:

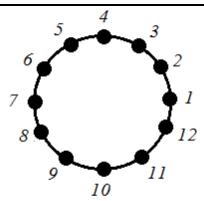
```
Program pppfff;
Label 1976,1977,1978,1979,1980;
Var beta,ksi,f,p,ekssentrisitet,l0,r,rb,nb,mb,ns,ms,nbz,mbz,pi,gamma:real;
```

```
Var ksisol,ksisag,ksiorta,zsol,zsag,zorta,epsr,roz,eb,k:real;
Var n,ks,i,j:integer;
Var es,fies,res,epsaxes,ass,radiuses,sigmaes:array[1..300] of real;
Procedure beton(n:integer; beta,ksi,k,epsr,gamma:real; var nbz,mbz:real);
Label 100,200,300,400,500;
Var i,j:integer;   var w1,w2,zj,h,f,nbz1,nbz2,mbz1,mbz2:real;
Begin    nbz1:=0;   nbz2:=0; mbz1:=0;  mbz2:=0;  if ksi>=2  then   goto 100;
if  (ksi<2)  and (ksi>=1+gamma)  then  goto 200;
if  (ksi<1+gamma)  and  (ksi>1-gamma)  then  goto 300;
if  (ksi>0)  and  (ksi<=1-gamma) then  goto 400;
100:  h:=2/n;  for  j:=1  to  n-1  do   begin
zj:=-1+j*h; w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(1-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2;
nbz1:=nbz1+h*f;  mbz1:=mbz1+h*f*zj;      end;
h:=2*gamma/n;  for  j:=1  to  n-1  do  begin
zj:=-gamma+j*h;  w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(gamma*gamma-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2; nbz2:=nbz2+h*f;  mbz2:=mbz2+h*f*zj;
end;      nbz:=nbz1-nbz2;  mbz:=mbz1-mbz2;   goto 500;
200:h:=ksi/n;  for  j:=1  to  n-1  do   begin
zj:=1-ksi+j*h; w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(1-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2;
nbz1:=nbz1+h*f;  mbz1:=mbz1+h*f*zj;    end;
h:=2*gamma/n;  for  j:=1  to  n-1  do  begin
zj:=-gamma+j*h;  w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(gamma*gamma-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2; nbz2:=nbz2+h*f;  mbz2:=mbz2+h*f*zj;
end;      nbz:=nbz1-nbz2;  mbz:=mbz1-mbz2;   goto 500;
300: h:=ksi/n;  for  j:=1  to  n-1  do   begin
zj:=1-ksi+j*h; w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(1-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2; nbz1:=nbz1+h*f;  mbz1:=mbz1+h*f*zj;
end;      h:=(gamma+ksi-1)/n;  for  j:=1  to  n-1  do  begin
zj:=1-ksi+j*h;  w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(gamma*gamma-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2; nbz2:=nbz2+h*f;  mbz2:=mbz2+h*f*zj;
end;      nbz:=nbz1-nbz2;  mbz:=mbz1-mbz2;   goto 500;
400: h:=ksi/n;  for  j:=1  to  n-1  do   begin
zj:=1-ksi+j*h; w1:=beta/ksi*(ksi-1+zj); w2:=sqrt(1-zj*zj);
f:=(k*w1-w1*w1)/(1+(k-2)*w1)*w2; nbz1:=nbz1+h*f;  mbz1:=mbz1+h*f*zj;
end;      nbz:=nbz1;  mbz:=mbz1;     500:end;
Procedure armatura(ks:integer; beta, ksi,epsr:real; es, fies, res, epsaxes,ass, radiuses:array[1..300] of real; var
sigmaes:array[1..300] of real; var ns,ms:real);
Label 1000,2000,3000;
Var i,j:integer;    Var epsilon,qq:real;
Begin   ns:=0;  ms:=0;      For  j:=1  to  ks   do    begin
epsilon:=beta*epsr/ksi*(ksi-1+radiuses[j]*sin(fies[j])/r);
if  abs(epsilon)<=epsaxes[j]   then  begin sigmaes[j]:=es[j]*epsilon; goto 1000; end;   if epsilon> epsaxes[j]
then  begin  sigmaes[j]:=res[j] ;  goto 1000; end;
if epsilon<-epsaxes[j]   then   begin  sigmaes[j]:=-res[j];  goto 1000; end;
1000:end;    for    j:=1  to  ks   do   begin  ns:=ns+sigmaes[j]*ass[j];
ms:=ms+ sigmaes[j]*ass[j]*radiuses[j]*sin(fies[j]);     end;  end;
begin r:=0.8; gamma:=0.5;  ks:=12;   pi:= 3.1415926535897932384626433 8;
for   j:=1  to   ks  do   begin   ass[j]:=0.00038013;  radiuses[j]:=0.769;
res[j]:=350000; es[j]:=200000000; epsaxes[j]:=res[j]/es[j];
fies[j]:=2*pi/ks*(j-1);   end; rb:=14500;  ekssentrisitet:=0.01; l0:=2;
epsr:=0.002;  roz:=epsr*l0/pi/pi/r;
eb:=27000000;  k:=eb*epsr/rb; n:=100;
writeln('Eb=',eb:12:2,'       Rb=',rb:12:2,'       gamma=',gamma:10:5,'       l0=',l0:8:3);
writeln('ekssentrisitet=',ekssentrisitet:10:5,'       R=',r:8:4);
beta:=0.1;1980:ksisol:=0.1;
```

```
beton(n,beta,ksisol,k,epsr,gamma,nbz,mbz);
armatura(ks,beta, ksisol,epsr,es, fies, res, epsaxes,ass, radiuses,sigmaes,ns,ms);
zsol:=2*r*r*r*rb*mbz+ms-(ekssentrisitet+roz*beta/ksisol)*(2*r*r*rb*nbz+ns);
1976:ksisag:=ksisol+0.3;  beton(n,beta,ksisag,k,epsr,gamma,nbz,mbz);
armatura(ks,beta, ksisag,epsr,es, fies, res, epsaxes,ass, radiuses,sigmaes,ns,ms);
zsag:=2*r*r*r*rb*mbz+ms-(ekssentrisitet+roz*beta/ksisag)*(2*r*r*rb*nbz+ns);
if zsol*zsag>0  then   begin  ksisol:=ksisag;  zsol:=zsag;  goto 1976; end;
1978:if  abs(ksisag-ksisol)<=0.00001   then   goto 1977;
ksiorta:=(ksisol+ksisag)/2;    beton(n,beta,ksiorta,k,epsr,gamma,nbz,mbz);
armatura(ks,beta, ksiorta,epsr,es, fies, res, epsaxes,ass, radiuses,sigmaes,ns,ms);
zorta:=2*r*r*r*rb*mbz+ms-(ekssentrisitet+roz*beta/ksiorta)*(2*r*r*rb*nbz+ns);
if  zsol*zorta>0   then   begin  ksisol:=ksiorta; zsol:=zorta;  goto 1978; end   else
begin ksisag:=ksiorta; zsag:=zorta;  goto 1978; end;
1977:ksi:=(ksisol+ksisag)/2;   beton(n,beta,ksi,k,epsr,gamma,nbz,mbz);
armatura(ks,beta, ksi,epsr,es, fies, res, epsaxes,ass, radiuses,sigmaes,ns,ms);
nb:=2*r*r*rb*nbz;  mb:=2*r*r*r*rb*mbz; f:=roz*beta/ksi; P:=nb+ns;
writeln('       beta=',beta:8:4,'      ksi=',ksi:8:4);
writeln('f=',f:12:5,'           P=',p:12:5);
for  j:=1  to   ks   do   writeln('sigmaes[',j:4,']=',sigmaes[j]:10:3);
writeln('****************************************');
beta:=beta+0.1;   if   beta<1.8  then  goto 1980;    end.
```

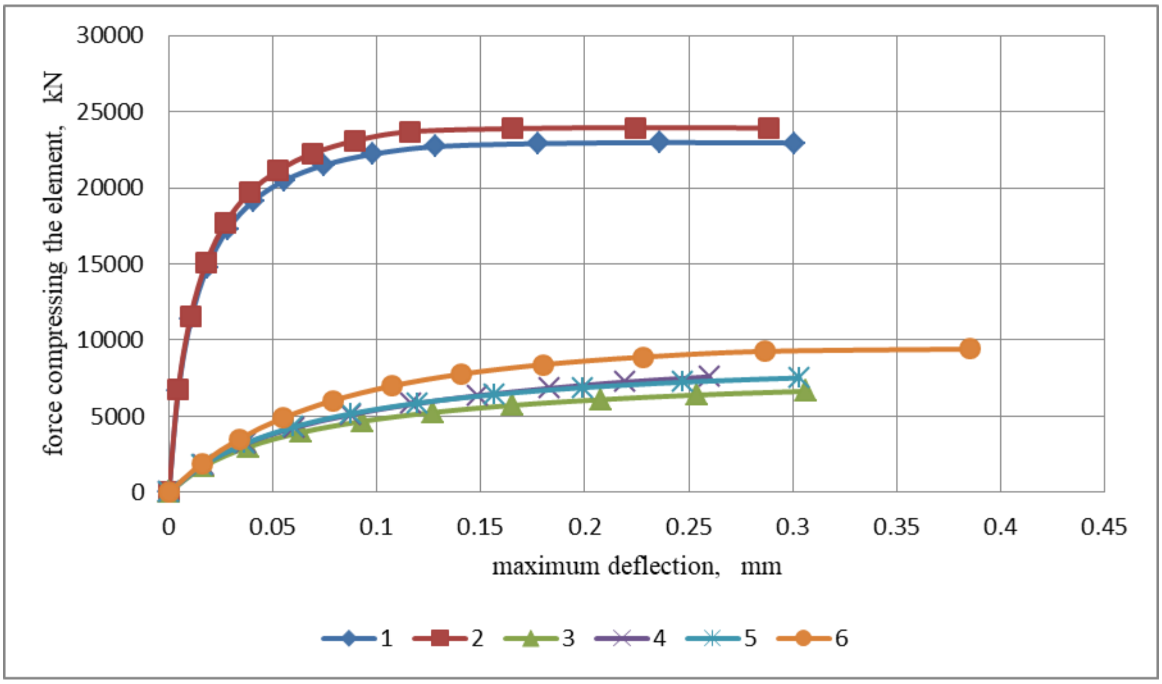
Numerical experiments. Based on the developed program module, numerical experiments were conducted for the case of conditional central compression, that is, the eccentricity of the compressive force $e = 1{,}0\ sm$. Initial data: $R_b = 14{,}5\ MPa$, $R = 0{,}8\ m$, $\gamma = 0{,}5$, $l_0 = 2\ m$, $k_s = 12$, $12\varnothing 22$, $A_{sj} = 3{,}8013\ sm^2$. The results of the calculations are given in Table 1 below. For the sake of clarity, the "load-deflection" graphs are given in Graph 1 below, and the graphs of the change in the stress of the reinforcement rods depending on the level of deformation are given in Graph 2. In order to study the effect of the reinforcement percentage, the case of reinforcing the section with $12\varnothing 28$ reinforcement was also investigated, provided that all other parameters were kept the same. As a result of these calculations, the corresponding graphs were constructed in Figure 1. The results of the calculations showed that as a result of increasing the total reinforcement area in the cross section from $A_s = 45{,}62\ sm^2$ to $A_s = 73{,}89\ sm^2$, the load-bearing capacity of the support increases from $P_{ult} = 22970{,}4\ kN$ to $P_{ult} = 23956{,}4\ kN$ qədər, that is, by only 4,3%. In order to study the effect of concrete class, calculations were performed for different classes of concrete while keeping the geometric parameters the same. Some of the results of these calculations are shown in Figure 1. It is clear from these graphs that the effect of concrete grade on the load-bearing capacity of compressed short supports is stronger than the increase in the area of reinforcement.

Table 1.



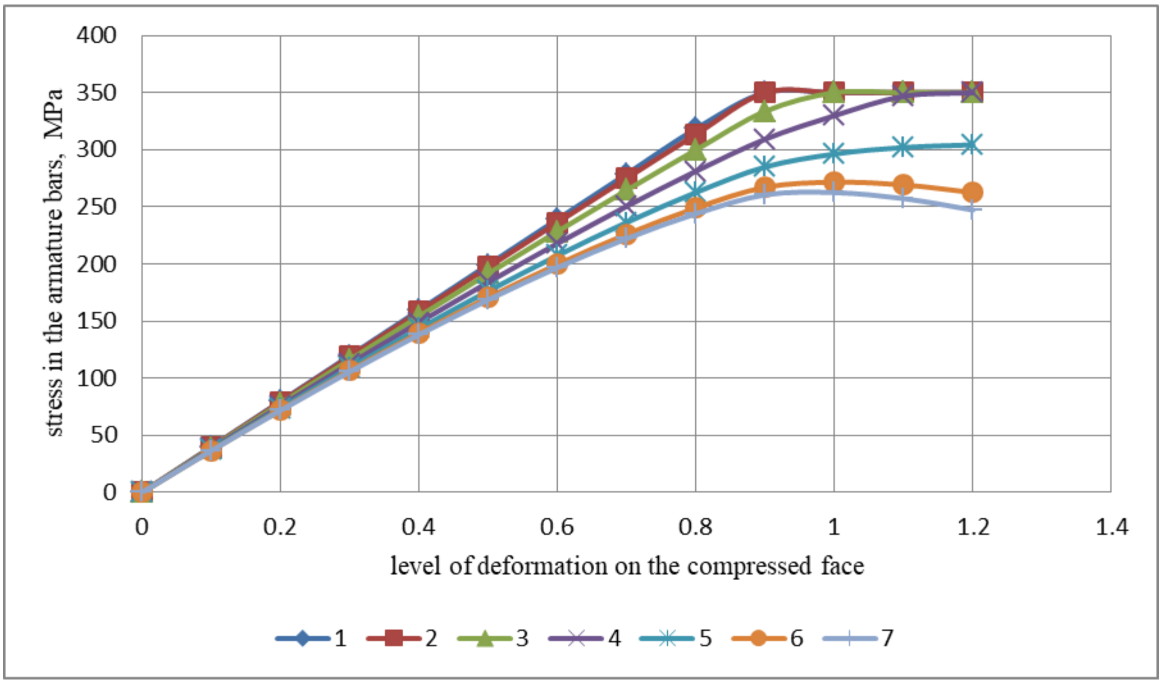
β	ξ	f	P	σ_4	$\sigma_3=\sigma_5$	$\sigma_2=\sigma_6$	$\sigma_1=\sigma_7$	$\sigma_8=\sigma_{12}$	$\sigma_9=\sigma_{11}$	σ_{10}
		mm	*kN*	*MPa*	*MPa*	*MPa*	*MPa*	*MPa*	*MPa*	*MPa*
0,1	22,16	0, 00457	6671,9	39,9	39,7	39,1	38,2	37,3	36,7	36,5
0,2	19,03	0, 01065	11370,3	79,8	79,3	77,8	75,8	73,8	72,3	71,8
0,3	16,49	0, 01844	14769,1	119,7	118,8	116,2	112,7	109,2	106,7	105,7
0,4	14,36	0, 02823	17267,4	159,6	158,1	154,2	148,9	143,5	139,6	138,1
0,5	12,53	0, 04044	19117,2	199,4	197,3	191,7	184,0	176,4	170,7	168,7
0,6	10,93	0, 05564	20485,8	239,2	236,3	228,6	218,0	207,5	199,8	196,9
0,7	9,52	0, 07454	21489,5	278,9	275,1	264,7	250,6	236,4	226,1	222,3

0,8	8,27	0, 09805	22212,2	318,5	313,5	299,9	281,3	262,7	249,1	244,1
0,9	7,11	0, 12833	22704,3	350	350	333,7	309,3	285,0	267,2	260,6
1,0	5,71	0, 17744	22903,6	350	350	350	329,9	296,3	271,6	262,6
1,1	4,72	0, 23591	22970,4	350	350	350	346,9	302,1	269,3	257,3
1,2	4,05	0, 30053	22947,7	350	350	350	350	304,3	262,6	247,3



Graph 1. The "load-deflection" graph of the support.

1- concrete $B\,20$, reinforcement $12\varnothing 22$, outer diameter $R = 0{,}8\ m$, inner diameter $r = 0{,}4\ m$;
2 - concrete $B\,20$, reinforcement $12\varnothing 28$, outer diameter $R = 0{,}8\ m$, inner diameter $r = 0{,}4\ m$;
3- concrete $B\,20$, reinforcement $12\varnothing 22$, outer diameter $R = 0{,}4\ m$, inner diameter $r = 0{,}2\ m$;
4 - concrete $B\,20$, reinforcement $12\varnothing 28$, outer diameter $R = 0{,}4\ m$, inner diameter $r = 0{,}2\ m$;
5- concrete $B\,30$, reinforcement $12\varnothing 22$, outer diameter $R = 0{,}4\ m$, inner diameter $r = 0{,}2\ m$;
6- concrete $B\,40$, reinforcement $12\varnothing 22$, outer diameter $R = 0{,}4\ m$, inner diameter $r = 0{,}2\ m$.



Graph 2. Variation of stresses in the reinforcement rods in the most stressed section depending on the deformation level parameter β .

1 - σ_4, 2 - $\sigma_3 = \sigma_5$, 3 - $\sigma_2 = \sigma_6$, 4 - $\sigma_1 = \sigma_7$, 5 - $\sigma_8 = \sigma_{12}$, 6 - $\sigma_9 = \sigma_{11}$, 7 - σ_{10} .

Main results:

1. It is shown that the load-bearing capacity of short reinforced concrete elements with annular cross-sections is mainly determined by the strength condition.
2. At the moment of loss of load-bearing capacity, the yield strength is not reached with all the reinforcement.
3. The effect of the concrete class on the load-bearing capacity is sufficient.
4. Since the element works mainly in compression, the load-bearing capacity of the element increases significantly with an increase in the diameter of the cross-section, for example, with the same reinforcement and the same class of concrete, doubling the diameter of the cross-section leads to an increase in the load-bearing capacity by approximately 3.53 times.
5. When reinforcing the element with 12 pieces of $\varnothing 22$ and $\varnothing 28$ reinforcements, with the same radius of the cross section and the same class of concrete, the load-bearing capacity increases by only 12.1%.
6. When studying the stress-strain state and load-bearing capacity of reinforced concrete elements with annular cross-sections working in compression, it is important to use realistic nonlinear deformation diagrams of materials.

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