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PARAMETRIC VIBRATIONS OF A VISCOUS ELASTIC MEDIUM – CONTACTING DAMAGED ORTHOTROPIC CYLINDRICAL SHELL STIFENED WITH INHOMOGENEOUS RINGS

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Abstract. In the paper we consider parametric vibrations of an viscous-elastic medium-contacting, damaged cylindrical shell stiffened in circular direction with inhomogeneous rings and subjected to the external action $p = p_0 + p_1 sin\omega_1 t$ (here, p_0 isan average or basic force, p_1 is the amplitude of force change , ω_1 is the frequency of change of the variable part of the force). The material of the cylindrical shell is considered orthotropic , the material of rings as inhomogeneous. The medium was modelled in the viscous-elastic form. An equation for finding critical force by means of contact conditions was structured and was studied depending on mechanical and geometrical parameters characterizing the system. Hereditary type damage formed due to vibrations for taking into account damages formed due to vibtations in the structure of the cylindrical shell subjected to the action of the external force.

Keywords: *viscoelastic medium, damaged cylindrical shell, inhomogeneous ring, parametric vibration*

To study a problem of parametric vibrations of an orthotropic, damaged, visco-elastic medium-contacting cylindrical shell subjected to the action of variable external force and stiffened with inhomogeneous rings, we use the Hamelton-Ostrogradsky variation principle (Fig.1.)

For total energy of the system under consideration we can write:

$$W = J + A_0 + A_1 + J_j \qquad (1)$$

Here J is the total energy of the cylindrical shell with changes taken into account [1], J_j is the total energy of inhomogeneous rings fastened to the cylindrical shell in annular direction [1], A_0 is the work done by the force acting on the cylindricak shell as viewed from the medium in the displaments of the shell, A_1 is the work done by the force p acting in the surface of the cylindrical shell in the displacements of the points of the shell. Let us write the expressions of quantities included in expression (1). For taking into account the damages, we use the hereditary type damage theory.

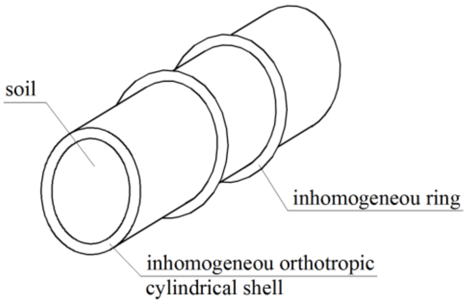

Fig. 1. A damaged, orthotropic viscoelastic medium contacting stiffened cylindrical shell with inhomogeneous rings

According to this theory, strain components in an inhomogeneous body are determined as follows [2]:

$$\varepsilon_{ij} = \bar{\varepsilon}_{ij} + M^* \cdot \sigma_{ij} \quad (2)$$

Here M^* is a heredity type integral operator describing the damage process, and is in the following form :

$$M^* \cdot \sigma_{ij} = \sum_{k=0}^{n} f(t_k^+) \int_{t_k^-}^{t_k^+} M(\bar{x}, t_k^+ - \tau) \cdot \sigma_{ij}(\tau) d\tau + \int_{t_{n+1}^-}^{t} M(\bar{x}, t - \tau) \cdot \sigma_{ij}(\tau) d\tau \quad (3)$$

In the expression (3) $M(\bar{x}, t-\tau)$ is a damage kernel, $(t_k^-; t_k^+)$ is time interval influenced by the active stress that increses the damages, $f(t_k^+)$ is a defect recovery function dependent on the volume of damages accumulated in one cycle. The value $f(t_k^+) = 0$ of this function corresponds full recovery of damages accoumulated in in cycle, the value $f(t_k^+) = 1$ corresponds to the falure of damage recovery process. The values between zero and a unit exprses partial recorery process. The values between zero and a unit expresses partial recovery of damages. A special condition should be given to determine the interval $(t_k^-; t_k^+)$.

This condition depends on the specific features of the construction, its operatig mode and the type of loading. The work A_0 done by the force acting on the cylindrical shell in the displacements of the shell. The work A_1 done by the force p acting on the surface of the cylindrical shell in the displacements of the points of the shell is determined as follows:

$$A_0 = -\int_{x_1}^{x_2} \int_{0}^{2\pi} q_z w dx dy \qquad A_1 = -4\int_{x_1}^{x_2} \int_{0}^{\pi/4} p w dx dy \quad (8)$$

The force included in the expression (8) and acting on the cylindrical shell as viewed from the medium is determined as follows:

$$q_z = k_\upsilon w - k_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - \int_0^t \Gamma(t-\tau) w(\tau) d\tau \quad (9)$$

Here k_ϑ is a Winkler coefficient, k_p is a Pasternak coefficient to be found experimentally, t is time, $\Gamma(t-\tau)$ is a viscosity kernel. Within the boundary conditions .

$\xi = 0$ and $\xi = \xi_1$ $\vartheta = w = T_1 = M_1 = 0$ we will look for the displacements of the shell as follows $(\xi = \frac{x}{R}; \xi_1 = \frac{l}{R})$:

$$u = u_0 \cos n\varphi \cos \frac{m\pi}{\xi_1} \xi \sin \omega t; \ \vartheta = \vartheta_0 \sin n\varphi \sin \frac{m\pi}{\xi_1} \xi \sin \omega t; w = w_0 \cos n\varphi \sin \frac{m\pi}{\xi_1} \xi \sin \omega t \quad (10)$$

Here u_0, ϑ_0, w_0 are unknown constants .

Using the solutions (10), the expressions, (8), (9) we can calculate the work done by the force p acting in the shell surface as viewed from the medium in the displacement points of the shell. By means of the approximations (10) we can determine active loading time included in damage operator from the decreasing condition of these functions:

$$\left[\left(\frac{\pi}{2}+2\pi k\right)\Big/\omega;\left(\frac{3\pi}{2}+2\pi k\right)/\omega\right].$$

The characteristic time T is determined as the largest among the times t_n^+ Taking into account the expressions (12), (13), having substituted (11) in (1) , integrating from $x_1=0$ to $x_2=l$ from $y_1=0$ to $y_2=2\pi$, from $t_0=0$ to $t_1=T$ for the Hamilton action $W_h=\int_{t_0}^{t_1}Wdt$ we obtain the expression $M(\bar{x},t-\tau)=\gamma=const)$. Usin the Ostrogoradsky-Hamilton $\delta W=0$ stationarity condition action, we obtain the following system of inhomogeneous functions with respect to the constats u_0,ϑ_0,w_0 Having solved the obtained system, we obtain the constants u_0,ϑ_0,w_0 , and based on this for the displacements of the points of the shell we obtain :

$$u=\frac{\Delta_1}{\Delta}\cos n\varphi\cos\frac{m\pi}{\xi_1}\xi\sin\omega t;\ \vartheta=\frac{\Delta_2}{\Delta}\sin n\varphi\sin\frac{m\pi}{\xi_1}\xi\sin\omega t;\ w=\frac{\Delta_3}{\Delta}\cos n\varphi\sin\frac{m\pi}{\xi_1}\xi\sin\omega t \quad (11)$$

In (11) Δ is the prinsipal determinant of the obtained system, $\Delta_i(i=1,2,3)$ are auxiliary determinants.

To determine the critical force, we use the equality $\frac{\Delta_3}{\Delta}=w_0$ As a result , we obtain:

$$p_{1h}=\left[\frac{w_0\Delta}{\alpha_{22}(\varphi_{11}\varphi_{22}-\varphi_{21}\varphi_{12})}-\frac{\alpha_{11}p_0}{\alpha_{22}}\right], \quad (12)$$

where

$$\alpha_{11}=\frac{4}{nk\omega}(coskL-1)sin\frac{n\pi}{4}(cos\omega T-1)$$

$$\propto_{22}=-\frac{2}{nk}\left(\frac{2}{\omega-\omega_1}sin(\omega-\omega_1)T-\frac{2}{\omega+\omega_1}sin(\omega+\omega_1)T\right)(coskL-1)sin\frac{n\pi}{4}$$

Changing the wave number n we calculate p_{1h} , and chosing $(p_{1h})_{min}$ among them we find the critical force p_{1hb} . When there are no damages, in expression $F(T)=\frac{1}{2\omega}\left(sin^2\omega T+4R_t sin^2\frac{\omega T}{2}\right)$ taking $R_t=0$ we can calculate the critivcal value of the force p_{1hb}. The expression (12) was numerically calculated for the following values of variables :

$$E_j=6,67\cdot10^9\frac{H}{M^2};\ R=160\,mm;\ h=0,45mm;\ \nu_1=0,11;\ \nu_2=0,19;\ L=800\,mm;$$

$$k_\vartheta=10^6\,N/m^3,\ \ k_p=10^4\,N/m;\ h_j=1,39mm;\ w_0=0,1mm;\ \omega=100\,hs;\ I_{kp.j}=0,48\,mm^4;$$

$$I_{xj}=19,9\,mm^4$$

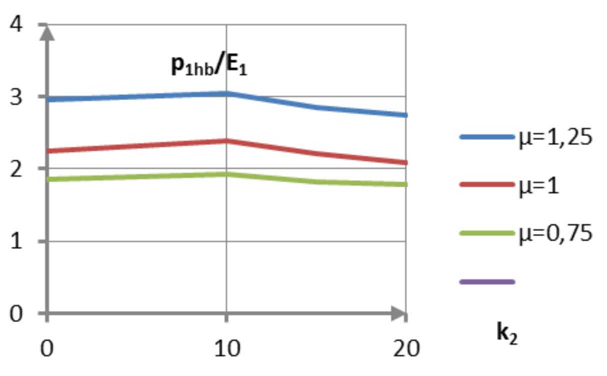

Fig 2. Dependence of the critical force on the amount of rings.

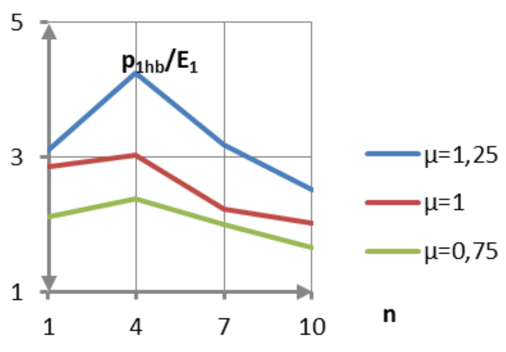

Fig.3. Dependence of the critical force on the wave number

The results of calculations are given in fig. 2 and fig. 3 in the form of dependence of $p_{1hb} / E_1 -$ on the amount of rings k_2, dependence of the amount of waves in cricular direction on n for various values of the ratio $\mu = \frac{E_1}{E_2}$. As can be seen form fig 2, with increasing k_2 the amount of ribs, at first critical force increases, and after some value begins to decrease.

This is explained by the fact that as the number of ribs increases, their mass also increases, and ultimately the effects of inertia on the vibration process become stronger. At the same time, as the orthotropy property of the shell material increases the value of the critical force also increases. Fig. 3 shows that as the number of waves in anunular direction increases, the increases, the critical force also increases, takes maximum value, again decreases, and its value approaches the critical value corresponding to the unstiffaned cylindrical shell

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