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INVESTIGATION OF THE STABILITY OF THE WALLS OF HORIZONTAL WELLS

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Abstract. *In presented work determine the dependence of the allowable wellhead pressure during the drilling of horizontal wells on the depth of drilling and the mechanical properties of the rocks, which ensures the stability of the walls of the wells and the circulation of the drilling fluid.*

Keywords: *Stress, pressure, isotropic soil, equilibrium equations, stress-strain state, borehole wall stability.*

It is known that successful drilling of deep horizontal oil and gas wells is closely related to the solution of the problem of rock stability around the well. The greater the depth of the drilled well, the greater the likelihood of accidents due to the loss of stability of the rocks around the wellbore. The liquidation of accidents leads to additional technological operations and a decrease in the technical and economic indicators of the well. Solving the problem of ensuring the stability of the well wall is one of the main tasks that drillers need to solve in order to achieve their goals. The solution to this problem lies in the definition of methods for predicting and preventing deformations that form on the borehole wall. In this work, given that the rocks around the wellbore are isotropic, we calculate the strength of the borehole wall. Let us assume that the inner radius of a horizontally drilled well r_0 is considered as a cylindrical object with an infinitely large outer radius (Fig. 1). In this case, the generalized Hooke's law in a cylindrical coordinate system for isotropic bodies has the following form [1];

$$\left.\begin{aligned}\varepsilon_r &= \frac{1}{E}\left(\sigma_r - \nu\sigma_\varphi - \nu\sigma_z\right)\\ \varepsilon_\varphi &= \frac{1}{E}\left(\sigma_\varphi - \nu\sigma_r - \nu\sigma_z\right)\\ \varepsilon_z &= \frac{1}{E}\left(\sigma_z - \nu\sigma_r - \nu\sigma_\varphi\right)\end{aligned}\right\} \qquad (1)$$

here $\sigma_r, \sigma_\varphi, \sigma_z -$ are the main stresses, $\varepsilon_r, \varepsilon_\varphi, \varepsilon_z -$ relative deformations in the main directions, E- Young's modulus.

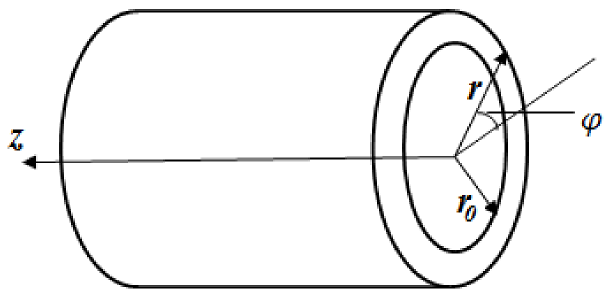

Fig.1

The system of equilibrium equations for the well-drilled soil massif is as follows [1];

$$\sigma_{ij,j} + \rho F_i = 0; \quad i,j = 1,2,3 \tag{2}$$

Here, j - differentiation according to the x_j coordinate;

the repetition of the index j means summing from one to three;

$F_i -$ is value of the projection of volume force in direction x_i.

Based on the principle of independence of the influence of forces, the stress state in the zone around the barrel is created due to two force factors, the wellhead pressure and the wight of the rock. So the current stresses can be expressed as follows;

$$\sigma_{ij} = \overline{\sigma}_{ij} + \sigma_{ij}^0 \tag{3}$$

where $\overline{\sigma}_{ij} -$ the stress due to the wellhead pressure,

$\sigma_{ij}^0 -$ is the stress due to the of weight.

The system of equilibrium equations for an intact soil mass under the influence of its own weight is as follows;

$$\sigma_{ij.j}^0 + \rho F_i = 0; \quad i,j = 1,2,3 \tag{4}$$

If to difer the equations (2) and (4) side by side we get

$$\overline{\sigma}_{ij,j} = 0; \quad i,j = 1,2,3 \tag{5}$$

If we represent the system of equations (5)in a cylindrical coordinate system, we get the following;

$$\frac{d\overline{\sigma}_r}{dr}+\frac{1}{r}\left(\overline{\sigma}_r-\overline{\sigma}_\varphi\right)=0 \tag{6}$$

Hooke's law for the stress state from wellhead pressure will look like this;

$$\left.\begin{aligned}\overline{\varepsilon}_r&=\frac{1}{E}\left[\overline{\sigma}_r-\nu\left(\overline{\sigma}_\varphi+\overline{\sigma}_z\right)\right]\\ \overline{\varepsilon}_\varphi&=\frac{1}{E}\left[\overline{\sigma}_\varphi-\nu\left(\overline{\sigma}_r+\overline{\sigma}_z\right)\right]\\ \overline{\varepsilon}_z&=\frac{1}{E}\left[\overline{\sigma}_z-\nu\left(\overline{\sigma}_\varphi+\overline{\sigma}_z\right)\right]\end{aligned}\right\} \tag{7}$$

As can be seen from (5), since there are no body forces, the stress state caused by wellhead pressure is a plane deformation state. In this case, $\overline{\varepsilon}_z=0$. Then from the third equation (7);

$$\overline{\sigma}_z=\nu\left(\overline{\sigma}_\varphi+\overline{\sigma}_r\right) \tag{8}$$

Then, taking into account (8) in (7) and after a series of transformations, the following system of equations is obtained;

$$\left.\begin{aligned}\overline{\varepsilon}_r&=\frac{1}{E}\left(a\overline{\sigma}_r-b\overline{\sigma}_\varphi\right)\\ \overline{\varepsilon}_\varphi&=\frac{1}{E}\left(a\overline{\sigma}_\varphi-b\overline{\sigma}_r\right)\end{aligned}\right\} \tag{9}$$

hear $a=1-\nu^2; b=\nu+\nu^2$.

If we solve (9) for $\overline{\sigma}_r$ and $\overline{\sigma}_\varphi$;

$$\left.\begin{aligned}\overline{\sigma}_r&=\frac{E}{a^2-b^2}\left(a\overline{\varepsilon}_r+b\overline{\varepsilon}_\varphi\right)\\ \overline{\sigma}_\varphi&=\frac{E}{a^2-b^2}\left(b\overline{\varepsilon}_r+a\overline{\varepsilon}_\varphi\right)\end{aligned}\right\} \tag{10}$$

Let us denote the components of the displacement vector from wellhead pressure as u, v, w .Here $u-$ are radial, $v-$ tangential, $w-$ longitudinal displacements. Since the task is symmetrical about the axis, the dependence of the relative deformations on the radial displacement u is as follows [1];

$$\left.\begin{aligned}\overline{\varepsilon}_r&=\frac{du}{dr}\\ \overline{\varepsilon}_\varphi&=\frac{u}{r}\end{aligned}\right\} \tag{11}$$

After several mathematical transformations, we get the following second-order differential equation;

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=0 \qquad (12)$$

It is known that the general solution of equation (12) is as follows [3];

$$u=c_1r+c_2r^{-1} \qquad (13)$$

where c_1 and c_2 are integration constants determined from the boundary conditions given below;

$$\left.\begin{array}{lll} when & r=r_0 & \bar{\sigma}_r=-P \\ when & r=\infty & \bar{\sigma}_r=0 \end{array}\right\} \qquad (14)$$

where P is the wellhead pressure.

In this case, since the stress state created in the borehole wall is a plane-deformed stress state, deformations occur only due to the pressure on the wellhead.

$$\left.\begin{array}{l} \bar{\sigma}_r=-P\dfrac{r_0^2}{r^2} \\ \bar{\sigma}_\varphi=P\dfrac{r_0^2}{r^2} \end{array}\right\} \qquad (15)$$

It is known that at depth H each element of the soil mass is subjected to all-round compression, and the stress state due to the own weight of the rock is defined as:

$$\sigma_r^0=\sigma_\varphi^0=\sigma_z^0=-\rho gh=-\gamma_q h \qquad (16)$$

where $\gamma_q -$ is the specific gravity of the rock.

Considering (15) and (16) in (3);

$$\left.\begin{array}{l} \sigma_r=-P\dfrac{r_0^2}{r^2}-\gamma_q h \\ \sigma_\varphi=P\dfrac{r_0^2}{r^2}-\gamma_q h \\ \sigma_z=-\gamma_q h \end{array}\right\} \qquad (17)$$

It is known that the stability condition for the first loading is as follows [2];

$$I_1^2+2(1+\nu)I_2=\sigma_c^2 \qquad (18)$$

where σ_c – is the stress yield for a given rock,

$I_1.I_2$ – are the first and second stress tensor invariants, respectively;

$$\left.\begin{array}{l} I_1 = \sigma_r + \sigma_\varphi + \sigma_z \\ I_2 = -(\sigma_r\sigma_\varphi + \sigma_r\sigma_z + \sigma_z\sigma_\varphi) \end{array}\right\} \tag{19}$$

It is known that the first plastic deformation occurs on the inner surface of the cylinder. When $r = r_0$, the stress state is defined as follows;

$$\left.\begin{array}{l} \sigma_r = -(P + \gamma_q h) \\ \sigma_\varphi = P - \gamma_{q_q} h \\ \sigma_z = -\gamma_q h \end{array}\right\} \tag{20}$$

Considering (20) in (19);

$$\left.\begin{array}{l} I_1 = -3\gamma_q h \\ I_2 = P^2 - 3\gamma_q^2 h^2 \end{array}\right\} \tag{21}$$

If we take into account (21) in (18);

$$2(1+\nu)P^2 + 3(1-2\nu)\gamma_q^2 h^2 - \sigma_c^2 = 0 \tag{22}$$

Let's solve equation (22) with respect to P;

$$P^2 = \frac{1}{2(1+\nu)}\left[\sigma_c^2 + 3(2\nu - 1)\gamma_q^2 h^2\right] \tag{23}$$

For the existence of real solutions of equation (23), the following condition must be satisfied;

$$\sigma_c^2 + 3(2\nu - 1)\gamma_q^2 h^2 \geq 0 \tag{24}$$

From (24) we determine the critical value of the well drilling depth for the given rock;

$$h_c = \frac{\sigma_c}{\gamma_q \sqrt{3(1-2\nu)}} \tag{25}$$

From (25) we can determine the crisis value of the wellhead pressure for the rock located at a depth that satisfies the condition $h < h_c$;

$$P_c = \sqrt{\frac{\sigma_y^2 - 3(1-2\nu)\gamma_q^2 h^2}{2(1+\nu)}} \tag{26}$$

It is known that Poisson's ratio is $\nu \approx 0{,}5$ when the deformation reaches the plasticity limit. In this case, from (26).

$$P_c = \frac{\sigma_y}{\sqrt{3}} \tag{27}$$

(27) coincides with the Mises condition.

Thus, for an isotropic rock located at a given depth, we determined the critical value of the wellhead pressure, which depends on the specific gravity of the rock. Mechanical characteristics of some rocks and other information are given in table 1

Table 1

Rock	yield point σ_y, Mpa	Jung's module E, 10^{-4} MPa	Poisson's number ν	Coefficient of linear expansion $\alpha, 10^6$ 1/c0	Special weight $\gamma, 10^{-4}$ n/m3	Pore e_0
sandy	50	2,35	0,25	10	2,5	0,2
lime	150	7,48	0,27	8	2,7	0,1
dolomite	245	8	0,25	7	2,9	0,25
gypsum	100	3,8	0,3	19	2,3	0,15
anhydride	170	3,8	0,3	19	2,9	0,45
clay	100	2	0,3	5,8	2,7	0,3

Table 2 shows the dependence of the wellhead pressure on the drilling depth for some rocks.

Table 2

H, 103 m	For rock P_b, MPa					
	sandy	lime	dolomite	gypsum	anhydride	clay
0,4	30,7	93,8	154,7			
0,5				61,84	104,97	61,33
0,6	29,4	93,4	154,36			
0,7				61	104,5	60,67

0,8	27,6	92,76	153,8			
0,9				60,4	103,93	59,78
1	25	92	153			
1,1				59,6	103,18	58,64
1,2	21	91	152,6			
1,3				58,6	102,27	57,25
1,4	16,3	89,9	151,7			
1,6	6,2	88,6	150,7			
1,63	0					
1,7				56,04	100	53,6
1,8		87	149,5			
2		85	148			
2,1				52,6	56,97	48,6
2,5		80	144	48,17	53,22	41,75
2,9				42,34	88,6	31,88
3				38,7	85,94	24,75
3,38						0
3,5		63,3	133,5	29,24	79,75	
3,97				0	72,2	
4		50,2	126,2			
4,1		46,91	122,9		67,75	
4,5		28,9	117,4		57	
4,7		10,4	112		50,4	
4,73		0				
4,8			111,28		42,38	
5			106,7			
5,35					0	
5,5			93,51			
6			76,45			

6,8			25,86			
6,898			0			

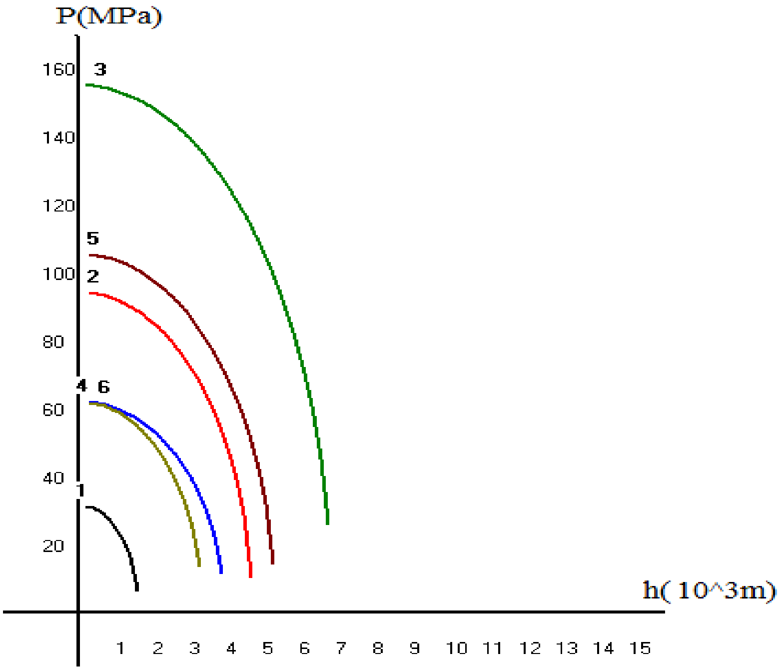

Fig. 2. Dependence of wellhead pressure crisis value on drilling depth for each rock: 1. sand, 2. lime, 3. dolomide, 4. gypsum, 5. anhydride, 6. clay.

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