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THE MAİN HYPOTHESES AND RELATİONS FOR DETERMİNİNG THE STRESS-STRAİN STATE ARİSİNG İN A PİPELİNE LAİD TO GREAT DEPTHS

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Abstract-*Oil, gas and oil products are transported through pipelines from where they are extracted or processed to where they are consumed. Pipelines cut and cross numerous small and large rivers, lakes and seas. For a long time, the transportation of hydrocarbons extracted from oil and gas fields in the Azerbaijani sector of the Caspian Sea has been carried out mainly with the help of underwater pipelines. Comparison of underwater pipelines with other vehicles used in the transportation of oil and oil products from the sea water area shows the superiority of pipelines.*

Keywords: *underwater pipelines, sea water area, sea currents, hydrological regimes, geological structure.*

There is a need to transport oil and gas produced in the sea to the shore and lay pipelines to great depths when the main pipelines pass through deep-water basins. At the initial stage, a long-length belt (lash) consisting of pipes is made.Then, when the length of the plate becomes greater than the depth of the water, they immerse it in water, filling the pipe with water by free sagging. At a later stage, most of the pipeline lies on the seabed, and the rest is suspended in the water by the tension applied at the end. Since the length of the pipeline section hanging in the water, laid at great depths, is much larger than the size of its cross-section, the pipeline can be modeled as a hemispherical rod lying at the bottom of the water and raised from one end to a height H above sea level. A part suspended in water is deformed under the action of the tension force N and gravity forces applied in the upper part. If we take into account that large relative displacements occur in such cases, then geometric nonlinearity should be taken into account when determining the stress-strain state that occurs in the pipeline.

Let's take the Cartesian coordinate system as shown below. The origin of the coordinates is located at the point of contact of the water bottom with the pipeline, the X axis is directed to the right in the horizontal direction, the Y axis is directed upward in the vertical direction (Fig.1.1).

It is assumed that the end of the pipeline with coordinates (L, 0) was raised to a height H before deformation. Where L is the length of the pipeline section that is suspended in water.

If we take into account the smallness of relative elongations and displacements in Cartesian coordinates, that is, Lagrange variables, until the equilibrium equations of the volume element are deformed, it will look like this [15].

$$\frac{\partial S_{mn}}{\partial x_n} + \rho F_m = 0 \quad . \qquad (1.1)$$

Here,

$$S_{mn} = \sigma_{nk}\left(\delta_{mk} + \frac{\partial u_m}{\partial x_k}\right) \qquad m, n, k = 1,2,3 \qquad (1.2)$$

corresponds to x1-x; x2-y; x3-z, summing from 1 to 3 on repeated indices;

σ_{nk} – components of the voltage tensor;

δ_{mk} – Kroneker symbols;

u_m – components of the displacement vector;

ρF_m – volume is the components of forces.

(1.1.) as can be seen from, its S_{mn} tensor is a non-symmetric Tensor.

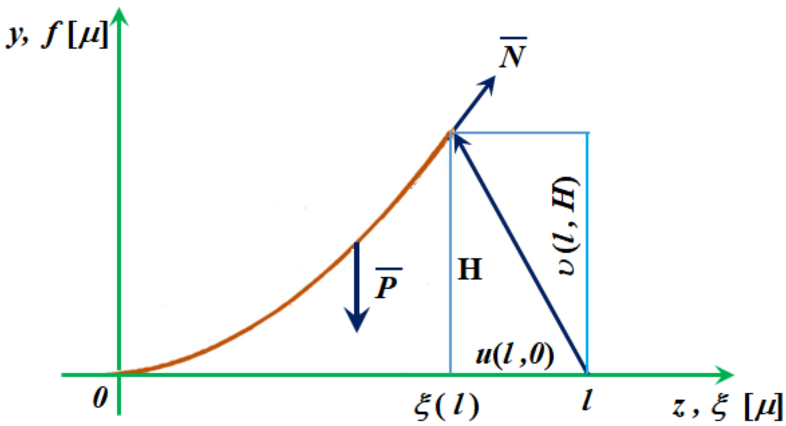

Figure 1.1. Semi-infinite with one end raised to H height schematic representation of the pipeline

The relationships between the components of tension and deformation tensors obey Huk's law [15].

$$\sigma_{nk} = \lambda\theta\delta_{uk} + 2\mu \cdot \varepsilon_{nk} . \qquad (1.3)$$

Here,

$$\theta = \varepsilon_{nk} \cdot \delta_{nk} , \qquad (1.4)$$

relative volume is variation.

λ, μ – Lame elasticity constants,

ε_{nk} - they are the components of the deformation Tensor.

The relationship between the deformation tensor and the displacement vector components is as follows:

$$\varepsilon_{nk} = \frac{1}{2}\left(\frac{\partial u_n}{\partial x_k} + \frac{\partial u_k}{\partial x_n} + \frac{\partial u_m}{\partial x_k}\cdot\frac{\partial u_m}{\partial x_n}\right), \qquad (1.5)$$

In the Equalities (1.4) and (1.5), there is also a summation on indices that are repeated from 1 to 3.

From the setting of the issue, it is clear that the pipeline bends in the plane of the Hoy, i.e.

$$u_3(x, y, z) \equiv 0. \qquad (1.6)$$

In accordance with x, directed along the y axes, and their displacement u_1 and u_2 does not depend on z, i.e.

$$\left.\begin{array}{l} u_1 = u_1(x, y) \\ u_2 = u_2(x, y) \end{array}\right\} \qquad (1.7)$$

Considering (1.6) and (1.7), we get from (1.3) and (1.5:

$$\left.\begin{array}{l} \varepsilon_{11} = \dfrac{1}{2}\left[2\dfrac{\partial u_1}{\partial x} + \left(\dfrac{\partial u_1}{\partial x}\right)^2 + \left(\dfrac{\partial u_2^2}{\partial x}\right)^2\right] \\ \varepsilon_{22} = \dfrac{1}{2}\left[2\dfrac{\partial u_2}{\partial y} + \left(\dfrac{\partial u_1}{\partial y}\right)^2 + \left(\dfrac{\partial u_2}{\partial y}\right)^2\right] \\ \varepsilon_{12} = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial y} + \dfrac{\partial u_2}{\partial x} + \dfrac{\partial u_1}{\partial x}\dfrac{\partial u_1}{\partial y} + \dfrac{\partial u_2}{\partial x}\dfrac{\partial u_2}{\partial y}\right) \\ \varepsilon_{33} = \varepsilon_{13} = \varepsilon_{23} = o \end{array}\right\}; \qquad (1.8)$$

$$\left.\begin{array}{l} \sigma_{11} = \lambda\theta + 2\mu\varepsilon_{11} \\ \sigma_{22} = \lambda\theta + 2\mu\varepsilon_{22} \\ \sigma_{12} = 2\mu\varepsilon_{12} \\ \sigma_{33} = \lambda\theta \\ \sigma_{13} = \sigma_{23} = 0 \end{array}\right\}; \qquad (1.9)$$

Considering (1.6) - (1.9) from (1.2):

$$\left.\begin{array}{l} S_{11} = \sigma_{11}\left(1 + \dfrac{\partial u_1}{\partial x}\right) + \sigma_{12}\dfrac{\partial u_1}{\partial y} \\ S_{12} = \sigma_{12}\left(1 + \dfrac{\partial u_1}{\partial x}\right) + \sigma_{22}\dfrac{\partial u_1}{\partial y} \\ S_{21} = \sigma_{11}\dfrac{\partial u_2}{\partial x} + \sigma_{12}\left(1 + \dfrac{\partial u_2}{\partial y}\right) \\ S_{22} = \sigma_{12}\dfrac{\partial u_2}{\partial x} + \sigma_{22}\left(1 + \dfrac{\partial u_2}{\partial y}\right) \\ S_{33} = \sigma_{33} \\ S_{13} = S_{31} = S_{23} = S_{32} = 0 \end{array}\right\}; \qquad (1.10)$$

Considering (1.9), the (1.1) equilibrium equations would be as follows:

$$\left.\begin{aligned}&\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{12}}{\partial y}+\rho F_x=o\\&\frac{\partial S_{21}}{\partial x}+\frac{\partial s_{22}}{y}+\rho F_y=o\end{aligned}\right\}. \qquad (1.11)$$

Considering that the length of the suspended section of the pipeline in water is much smaller than its cross-sectional dimensions when laying to great depths, it can be assumed that the sections that are perpendicular to the axis of the pipe before deformation remain perpendicular and plane to the axis even after deformation, that is, the driving is not taken into account, This means that its σ_{12} tension can be ignored in σ_{11} comparison with. The side faces, on the other hand, are free of force, that is, and their σ_{33} and σ_{22} tension is also very small compared to their σ_{11} tension. Onda from (1.10) and (1.11):

$$\left.\begin{aligned}&\frac{\partial}{\partial x}\left[\left(1+\frac{\partial u_1}{\partial x}\right)\sigma_{11}\right]=o\\&\frac{\partial}{\partial x}\left[\frac{\partial u_2}{\partial x}\sigma_{11}\right]-P=o\end{aligned}\right\}. \qquad (1.12)$$

Here, P is the volumetric weight of the material of the rod.

According to the principle that the effects of forces do not depend, the stresses arising due to the pressure of water are calculated separately and are added to the stresses arising from the action of specific gravity by the tensile force.

And now (1.12) let's take the equilibrium equations as the equilibrium of the element of the pipe with the length dx to deformation and ds after deformation. the ds element is influenced by the forces at the ends –T and T+dT , and in the middle-Fds.

$$\overline{F}ds+\left(-\overline{T}\right)+\overline{T}+d\overline{T}=0.$$

From here,

$$\overline{F}ds+d\overline{T}=0,$$

or

$$\overline{F}+\frac{d\overline{T}}{ds}=0, \qquad (1.13)$$

Let's write the equation (1.13) in projections ξ,η,ξ onto its axes. Considering that,

$$T_\xi=T\cdot\frac{d\xi}{ds};\,T_\eta=T\frac{d\eta}{ds};\,T_\xi=T\frac{d\zeta}{ds},$$

(1.13) from:

$$\left.\begin{aligned} &\frac{d}{ds}\left(T\frac{d\xi}{ds}\right)+F_{\xi}=o\\ &\frac{d}{ds}\left(T\frac{d\eta}{ds}\right)+F_{\eta}=o\\ &\frac{d}{ds}\left(T\frac{d\zeta}{ds}\right)+F_{\zeta}=o \end{aligned}\right\}, \tag{1.14}$$

Relationships between ξ,η,ζ Euler variables and Lagrangian variables x,y,z

$$\left.\begin{aligned} &\xi=x+u(x)\\ &\eta=\upsilon(x)\\ &\xi=w(x) \end{aligned}\right\}, \tag{1.15}$$

can be in shape.

Equilibrium equations for the case under consideration:

$$\left.\begin{aligned} &\frac{d}{ds}\left(T\frac{d\xi}{ds}\right)=o\\ &\frac{d}{ds}\left(T\frac{d\eta}{ds}\right)=q \end{aligned}\right\}. \tag{1.16}$$

Here,

$$\left.\begin{aligned} &d\left(T\frac{d\xi}{ds}\right)=0\\ &d\left(T\frac{d\eta}{ds}\right)=q_0dx \end{aligned}\right\}. \tag{1.17}$$

If we take into account that for the case $\zeta=w(x)\equiv o,$ under consideration we will get from the equality (1.15:

$$ds=\sqrt{(d\xi)^2+(d\eta)^{2'}}=\sqrt{(1+u')^2}\cdot dx.$$

Then the expressions $\frac{d\xi}{ds},\frac{d\eta}{ds}$ included in (1.17)

$$\left.\begin{aligned} &\frac{d\xi}{ds}=\frac{1+u'}{\sqrt{(1+u')^2+\upsilon^2}}\\ &\frac{d\eta}{ds}=\frac{\upsilon'}{\sqrt{(1+u')^2+\upsilon'^2}} \end{aligned}\right\}, \tag{1.18}$$

it takes shape.

$$d\left[(1+u')\frac{T}{\sqrt{(1+u')^2+\upsilon'^2}}\right]=0$$
$$d\left[\upsilon'\frac{T}{\sqrt{(1+u^1)^2+\upsilon'^2}}\right]=q_0dx \quad . \qquad (1.19)$$

Considering that,

$$u=u_1;\ \upsilon=u_2;\ \sqrt{(1+u'^2)+\upsilon'^2}=\frac{ds}{dx};$$
$$q_\circ=\rho\cdot u'=\frac{\partial u_1}{\partial x};\ \upsilon'=\frac{\partial u_2}{\partial x};\ \sigma_{11}=T\cdot\frac{dx}{ds}$$

(1.19) we will take from:

$$\left.\begin{aligned}&\frac{d}{dx}\left[\left(1+\frac{\partial u_1}{\partial x}\right)\sigma_{11}\right]=o\\&\frac{d}{dx}\left[\frac{\partial u_2}{\partial x}\sigma_{11}\right]=P\end{aligned}\right\} . \qquad (1.20)$$

$\frac{\partial}{\partial x}=\frac{d}{dx}$ considering that the (1.20) system coincides with the (1.12) system. Considering that Huk's law in Euler variables

$$T=Ee, \qquad (1.21)$$

More precisely than Hooke's law in Lagrange variables, it is more convenient to use equations (1.19) to study the stress-strain state of a part of a pipeline suspended in water during its laying on the seabed.

In (1.21), the E-yung module is the relative extension of the e-ds element, and

$$e=\frac{ds-dx}{dx}=\sqrt{(1+u'^2)+v'^2}-1 . \qquad (1.22)$$

Huk's law in Lagrangian variables is as follows for the case under consideration:

$$\sigma_{11}=E\varepsilon_{11}=\frac{E}{2}\left[2\frac{\partial u_1}{\partial x}+\left(\frac{\partial u_2}{\partial x}\right)^2\right] . \qquad (1.23)$$

In small deformations $\varepsilon_{11}\approx e$. However, in the case under consideration, ε_{11} deformations are finite and do not carry any physical meaning. Therefore, the Equality (1.21) is more accurate than the Equality (1.22).

$$u_1=u;\ u_2=v$$
$$\sigma_{11}=E\varepsilon_{11}=\frac{E}{2}\left[2\frac{\partial u}{\partial x}+\left[\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right], \qquad (1.24)$$

$$T=E\left[\sqrt{(1+u')^2+v^{12}}-1\right] . \qquad (1.25)$$

If we show its tension $\sigma_{11}(x,y)$ as the sum of its tension $\sigma_{11}(x,0)$ arising from the action of the tensile force N1 and regularly distributed along the cross-section and resulting from the bending of the pipeline and proportional to the variable y $\frac{E_y}{\rho}$

$$\sigma_{11}(x_1 y) = \sigma_{11}(x_1 o) - \frac{E}{\rho} y \quad , \qquad (1.26)$$

it is possible.

Here the minus sign indicates that the lower part of the pipe (y<0) is stretched, this is the radius of curvature of the pipeline axis. Then from (1,24)

$$\sigma_{11}(x_1 0) = E\varepsilon_{11}(x_1 0) = \frac{E}{2}\left[2u' + u'^2 + v'^2\right] . \qquad (1.27)$$

In the case of small deformations, the equalities (1.25) and (1.27) are reduced to the following:

$$T = E\left(\sqrt{1+2u'} - 1\right), \qquad (1.28)$$

$$\sigma_{11} = Eu' . \qquad (1.29)$$

If we build $\sqrt{1+ru'}$ an expression $u' = 0$ around it and settle for the linear part, we get:

$$\sqrt{1+ru'} = 1 + u' , \qquad (1.30)$$

If we write (1.30) instead of u in (1.28)

$$T = Eu' , \qquad (1.31)$$

it is possible.

Beləliklə kiçik deformasiyalar halında T(x,0) və $\sigma_{11}(x,0)$ tamamilə üst-üstə düşür. (1.22) bərabərliyindəki nisbi uzanma yerdəyişmələrin istənilən qiymətində fiziki mənaya malikdir. (1.23) bərabərliyinə daxil olan ε_{11} aşağıdakı kimi təyin olunur. [15,64]

$$\varepsilon_{11} = \frac{1}{2} \cdot \frac{(ds)^2 - (dx)^2}{dx^2} = \frac{ds - dx}{dx} \cdot \frac{ds + dx}{2 \cdot dx} , \qquad (1.32)$$

As can be seen from (1.32), when, that is, when the extensions are infinitesimal

$$\varepsilon_{11} \approx \frac{ds - dx}{dx} \approx e .$$

That is, in the case of infinitesimal deformations, e and ε_{11} coincide, but have no physical meaning, as can be seen from (1.32), when the deformations ε_{11} are finite.

In this half-year, let's note the main results that are important for students and engineers to know:

CONCLUSIONS

1. The problem was set in both LaGrange and Euler variables to determine the stress-deformation state of a pipeline that oscillates to great depths.

2. The equilibrium equations of the water-dependent part of the underwater pipeline were compiled in both LaGrange and Euler variables and their equivalence was proved.

3. At any value of displacements, geometric relationships are obtained that have a physical meaning and take into account nonlinearity.

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